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MA 530
Spring 2023

Homework 1

Due January 18th on paper at the beginning of class. Please let me know if you have a question or find a mistake.

- Taylor Section 1.1: # 12.
- Taylor Section 1.2: # 3 (only do $f(z) = e^{1/z}$ and $f(z) = e^{-|z|^2}$), # 5, #7.
- Taylor Section 1.3: # 2.
- # 4 from Section 1.1.4 (page 9) of Ahlfors' book:
  `https://www.matem.unam.mx/~hector/%5BLars_Ahlfors%5D_Complex_Analysis_(Third_Edition).pdf`
  In this problem, a, b, and c are given complex numbers and you are solving for z.

  Hint: One way to do this problem is this: first solve it in the case where a and b are real by taking real and imaginary parts of the equation and solving for $\operatorname{Re} z$ and $\operatorname{Im} z$. Then reduce to this case using a substitution $z = e^{i\varphi} w$ for a strategically chosen real number φ and then multiplying the equation through by $e^{i\psi}$ for another strategically chosen real number ψ. To find φ and ψ, take any real α and β such that $a = |a| \exp(i\alpha)$ and $b = |b| \exp(i\beta)$ and substitute these polar forms into the equation. The final condition on a and b has a simple form, but you do not need to simplify the resulting complicated formula for z; just write it out in terms of a, b, c, α, β.

  An alternative substitution, avoiding polar form and instead using square roots of complex numbers, is $z = a^p \bar{a}^q b^q \bar{b}^p w$, followed by multiplying through by $a^r b^r$ or $\bar{a}^r \bar{b}^r$, where p, q, r are strategically chosen multiples of $1/2$. Square roots of complex numbers can be found using polar form, or alternatively more directly as in Section 1.1.2 (page 3) of Ahlfors' book.

  Finally, as a last resort there is always the brute force method: take the real and imaginary parts of the equation, and solve the resulting system of two linear equations in two unknowns.