

Homework 2

Due January 25th on paper at the beginning of class. Please let me know if you have a question or find a mistake.

- Section 1.2: # 8, # 9 (for this one, replace $C(\Omega)$ by $C^1(\Omega)$), # 10.
- Section 1.5: # 1, # 2, # 7, # 12, # 21.
- Additional problems
 1. Given $z \in \mathbb{C} \setminus \{0\}$ and a specific value of $\arg z$, show that $\log z = \log |z| + i \arg z$ with that value of $\arg z$ is given by $\int_{\gamma} d\zeta/\zeta$, where γ is the curve consisting of the segment from 1 to $|z|$ followed by the arc from $|z|$ to z through angle $\arg z$.
 2. Write out the calculation of $\int_{\gamma} d\zeta/\zeta$ and sketch γ , with γ as above, for the case $z = 4i$ and $\arg z \in (-8\pi, -6\pi)$.

This is not part of the homework, but regarding the last two problems: each lap that γ makes around the origin corresponds to moving z to a neighboring sheet in the Riemann surface of the logarithm (i.e. up or down a level in the infinite parking garage.) In the last problem we travel to a sub-basement level. Here is a link with more discussion of the Riemann surface and a picture:

<https://ocw.mit.edu/courses/18-04-complex-variables-with-applications-fall-1999/pages/study-materials/riemann-surfaces-the-logarithm/>.