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MA 530
Spring 2023

Homework 3

Due February 1st on paper at the beginning of class. Please let me know if you have a question or find a mistake.

- Section 2.1: # 3, # 6, # 9,
- Section 2.2: # 1, # 3 (for this one, justify differentiating under the integral sign using Rudin's Theorem 9.42.[1]), # 4, # 12, # 13 (for this one, where it says '$\sup_{z\in\Omega}|f(z)| = |f(\zeta_0)| = Be^{i\alpha}$' it should say '$\sup_{z\in\Omega}|f(z)| = |f(\zeta_0)| = B$ and $f(\zeta_0) = Be^{i\alpha}$.').

For # 4, ignore the part that says 'Relax the hypothesis that f is bounded'. If you get stuck, try applying the maximum principle to f_ε on $\Omega_A = \{z \in \Omega \colon |\operatorname{Im} z| < A\}$, with A chosen large enough that the maximum is attained on $\partial\Omega_A \cap \partial\Omega$. It might also help to read the Wikipedia article on Phragmén–Lindelöf principles as far as the section "Outline of the technique". In this exercise you are following that outline. The "Example of application" in the article is a stronger version of this exercise, with suitably relaxed hypothesis.

[1]See `https://web.math.ucsb.edu/~agboola/teaching/2021/winter/122A/rudin.pdf` if you don't have it. Checking the hypotheses should be quick. If you're curious about the proof, note that it is short and only uses the mean value theorem and the fact that limits and integrals can be switched when convergence is uniform.