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MA 530
Spring 2023

Homework 5

Due March 1st on paper at the beginning of class. Please let me know if you have a question or find a mistake.

- Section 3.1: # 1, # 5.
- # 1 from Section 5.1.1 (page 178) of Ahlfors' book:

  `https://www.matem.unam.mx/~hector/%5BLars_Ahlfors%5D_Complex_Analysis_(Third_Edition).pdf`

  To solve this, first prove that if f_1, $f_2, \ldots$ is a sequence of continuous functions converging to f uniformly on a compact set K, then $e^{f_n(z)} \to e^{f(z)}$ uniformly on K. Second, use equation (1.5.37) from Taylor to prove that, for any $R > 0$, $n \log(1 + \frac{z}{n}) \to z$ uniformly on $\{z \in \mathbb{C} \colon |z| \leq R\}$.
- Additional problem. Let $f \colon \Omega \to \mathbb{C}$ be given by

  $$f(z) = \frac{\sin \frac{z}{2} \sin \frac{z}{3}}{\sin \frac{z}{5} \sin \frac{z}{7}},$$

  where $\Omega = \{z \in \mathbb{C} \colon \sin \frac{z}{5} \sin \frac{z}{7} \neq 0\}$. Find a point in $\mathbb{C}$ where f has a simple pole, a point where it has a double pole, a point where it has a simple zero, a point where it has a double zero, and a point where it has a removable singularity.

  Finally, you may like to (but are not required to) examine f near these points using `https://samuelj.li/complex-function-plotter`