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MA 530
Spring 2023

Homework 6

Due March 8th on paper at the beginning of class. Please let me know if you have a question or find a mistake.

- Section 3.2: # 2 and # 3, but replace # 2 by the following somewhat simpler problem:

  Let $t > 0$ be given, and let $f(x) = e^{-x^2/4t}$. Show that

  $$\frac{1}{\sqrt{2\pi}} \sum_{k=-\infty}^{\infty} \hat{f}(k) e^{ikx} = \sum_{\ell=-\infty}^{\infty} f(x + 2\pi\ell), \quad \text{where } \hat{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-iky} f(y) dy.$$

  Use the hints in the book, but for # 3 where it suggests $t = \pi\tau$ it should be $t = \pi/\tau$.

  Interpretation: When τ is small, the left side of (3.2.51) converges slowly and the right side converges quickly, so taking just a few terms on the right gives an accurate approximation to the full sum (whereas on the left we would have to take many terms). We will return to this later in the course. In the meantime, you may enjoy using Desmos to see how many more terms it takes to compute the integer part of $\sum_{n=0}^{\infty}(0.999)^{n^2}$ using the left side of (3.2.51) as opposed to the right side, and how the difference becomes more stark for 0.999999 etc until eventually Desmos can't handle it the naive way. For more on (3.2.51), see Section 1.7.5 of the book 'Fourier Series and Integrals' by Dym and McKean, and for an interpretation in terms of heat flow, see Section 1.8.3.

- Section 4.1: # 6 and # 1.

  Hints: Do # 6 first, and use that result to ease the computation of residues in the other problems. For the second of # 1, write in terms of complex exponentials and use even/odd functions to reduce it to one of the examples done in the chapter (no need to redo the residue argument then). For the third of # 1, use $\Omega_R = \{z \in \mathbb{C} \colon 1/R < |z| < R \text{ and } 0 < \arg z < 2\pi/3\}$.