

Homework 7

Due March 22nd on paper at the beginning of class. Please let me know if you have a question or find a mistake. Some of these problems reference Fisher's book:

<https://archive.org/details/complex-variables-2ed-dover-1999-fisher/page/n181/mode/2up>

- Taylor Section 4.1: # 12. (Additional hint: look at Example 8 from Fisher Section 2.6.)
- Fisher Section 3.1: # 12.
- Taylor Section 4.2: # 3.
- Additional Problems:
 1. Use the open mapping theorem (as stated in Taylor Proposition 4.2.8) to give a short proof of the maximum principle (as stated in the first two sentences Taylor Proposition 2.2.1 leading up to (2.2.5) – you can omit the last sentence containing (2.2.6)).
 2. Use series expansions to show that if f and g are holomorphic near a point p , f has a zero of order at least k at p , and g has a zero of order k at p , then

$$\lim_{z \rightarrow p} \frac{f(z)}{g(z)} = \frac{f^{(k)}(p)}{g^{(k)}(p)}.$$

3. Prove that if f has a pole of order k at p then the residue of f at p is given by taking $k - 1$ derivatives of $(z - p)^k f(z)/(k - 1)!$ and evaluating at $z = p$.