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## Homework 9

Due April 21st on paper at the beginning of class. Please let me know if you have a question or find a mistake.

• Fisher Section 3.5: #1 and #2.

*Hint:* For the second problem, it will be helpful to use a mapping of the form  $z \mapsto z^{\beta}$  for suitable  $\beta$ ; be careful to clearly explain what branch you are using, including the domain and range and how it is defined.

• Taylor Section 5.1: #2.

*Hint:* Use the result of the first problem above to reduce this to a problem in the upper half plane, where a mapping of the form  $w \mapsto aw + b$  for suitable constants a and b does the job.

• Fisher Section 3.4: #3.

*Hint:* There is a solution sketch in the back of the book, but be sure to fill in the details if you use it.

• Taylor Section 4.2: #5.

*Hint:* Use the result of # 3 from the same section, and mimic the proof of Proposition 4.2.10.

• Additional problem. Let  $\Omega \subset \mathbb{C}$  be open, let  $p \in \Omega$ , and let  $D = \{z \in \mathbb{C} : |z| < 1\}$ . Let  $f_1$  and  $f_2$  be biholomorphic maps from  $\Omega$  to D such that  $f_j(p) = 0$  and  $f'_j(p) > 0$  for each  $j \in \{1, 2\}$ . Use Example 4 from Fisher Section 3.4 to prove that  $f_1 = f_2$ .