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MA 562
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# Homework 3

Due September 16th on paper at the beginning of class. Please let me know if you have a question or find a mistake.

1. Let $A\colon \mathbb{R}^m \to \mathbb{R}^p$ and $B\colon \mathbb{R}^n \to \mathbb{R}^m$ be linear maps, $a = \operatorname{rank} A$, $b = \operatorname{rank} B$, $c = \operatorname{rank} AB$. For each of the following, state whether it is true or false. If true, give a proof and if false give a counterexample. For the proofs, use the facts that the rank of a linear map is the dimension of its image, and the rank of the corresponding matrix equals the rank of its transpose.
   (a) We always have $c \leq \min\{a, b\}$.
   (b) If $A$ is injective then $c = \min\{a, b\}$.
   (c) If $A$ is surjective then $c = \min\{a, b\}$.
   (d) If $B$ is injective then $c = \min\{a, b\}$.
   (e) If $B$ is surjective then $c = \min\{a, b\}$.
2. Find explicit examples of the maps $G$ and $H$ from the rank theorem (Theorem II.7.1) in the following cases:
   (a) $F$ is as in the example preceding Theorem II.7.1 and $a = (1, 0)$; note that in the first edition there is a misprint in the definition of $F$ and $(x^1)^2$ should be $(x^1)^2 + (x^2)^2$.
   (b) $F(x^1, x^2) = ((x^1)^2 + (x^2)^2, 2(x^1)^2 + 2(x^2)^2)$ and $a = (1, 0)$.
3. Exercise III.4.3
4. Exercise III.4.7
5. Exercise III.5.6
6. Find explicit preferred coordinates of the equator $z = 0$ as a submanifold of the unit sphere $\{(x, y, z) \in \mathbb{R}^3 \colon x^2 + y^2 + z^2 = 1\}$, near the point $(1, 0, 0)$.