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MA 562
Fall 2022

Homework 4

Due October 7th on paper at the beginning of class. Please let me know if you have a question or find a mistake.

1. Exercise III.6.9

   Hint: Use Theorem III.5.7. Note that there is a typo in the first line of Example III.6.15: where it says $H = G \circ F(\mathbb{R})$ it should be $H(\mathbb{R}) = F \circ G(\mathbb{R})$.

2. Exercise IV.6.4

3. For the actions in the above problem, find all the corresponding orbits in $\mathbb{R}^2$. Sketch a few representative orbits in the plane, and attach a few corresponding velocity vectors to each of the non-constant orbits in the manner of Figure IV.8.

4. Exercise 2 from page 2 of `https://www.math.purdue.edu/~kdatchev/562/ode.pdf`.

5. Problems 1 and 2 (and, if you can, Problem 3) from Section 1.18 (page 35) of Arnold's *Ordinary Differential Equations* book.[1]

   Hint: A great circle of a sphere is a circle which lies on a two-plane through the sphere's center. It can help to use complex variables, and consider the differential equations solved by $x^1 + ix^2$ and $x^3 + ix^4$.

[1] Available online at
`https://loshijosdelagrange.files.wordpress.com/2013/04/vladimir-i-arnold-vladimir-i-arnold-roger-cooke-ordinary-differential-equations-1992.pdf`.
For more on pendula, see pages mentioned in the index of that book. Sections 1.16–1.18 and 12.9 are particularly interesting and pertinent. That book has many more treasures: the hunter and rabbit example is on page 217, and section 33 has a beautiful introduction to manifolds, with lots of examples and pictures. Another good reference for the pendulum is section 30 of Poonen's ODE notes `https://math.mit.edu/~poonen/notes03.pdf`.