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MA 562
Fall 2022

Homework 5

Due October 14th on paper at the beginning of class. Please let me know if you have a question or find a mistake.

1. Let $F\colon \mathbb{R}^2 \to \mathbb{R}^2$ be given by $F(r,\theta) = (x,y) = (r\cos\theta, r\sin\theta)$. Find $\alpha\colon \mathbb{R}^2 \to \mathbb{R}$ and $\beta\colon \mathbb{R}^2 \to \mathbb{R}$ such that $F_*(r\partial_\theta) = \alpha(x,y)\partial_x + \beta(x,y)\partial_y$. Then find $F^*(\alpha(x,y)dx + \beta(x,y)dy)$.

2. Exercise V.1.5.

3. Exercise V.1.6.

   Hint: Use Example IV.2.5.

4. Exercise V.3.2. Give your answer in the form

$$g_{11}(\theta,\varphi)d\theta^2 + g_{22}(\theta,\varphi)d\varphi^2;$$

   note that the cross terms cancel.