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MA 562
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Homework 6

Due October 21st on paper at the beginning of class. Please let me know if you have a question or find a mistake.

1. Let $f(t) = e^{-1/t}$ when $t > 0$ and $f(t) = 0$ when $t \leq 0$. Prove that $f \in C^\infty(\mathbb{R})$.

   Hint: Prove that the set {functions g such that $g(t) = P(1/t)f(t)$ for some polynomial P} consists only of continuous functions and is closed under differentiation.

2. Exercise V.4.1. Note that the hint for Exercise V.4.2 is also useful here. Show by way of example that the conclusion is invalid if the word 'closed' is deleted.

3. Exercise V.5.1. Here there is a typo in the statement and the word 'show' should be inserted after the word 'and').

   Hint: For the positive statement, mimic the decomposition of a function into even and odd parts, or of a complex number into real and imaginary parts. For the negative statement, use a permutation of r elements with sign 1.

4. Exercise V.6.6 from page 207. Omit the last sentence.

   Hint: Let $\{\partial_{x^1}, \dots, \partial_{x^n}\}$ be a basis of V, and $\{dx^1, \dots dx^n\}$ the corresponding dual basis of V^*. Use linearity to reduce to the case where $v = \partial_{x^k}$, $\varphi = dx^{i_1} \wedge \cdots \wedge dx^{i_r}$, $\psi = dx^{i_{r+1}} \wedge \cdots \wedge dx^{i_{r+s}}$, and compute directly in that case.

5. Another interesting place to read about differential forms is Lorenzo Sadun's notes `https://arxiv.org/pdf/1604.07862.pdf`. The material there is presented in a different order from ours but very lucidly. Do Exercise 6 from page 7.