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MA 562
Fall 2022

Homework 8

Due November 18th on paper at the beginning of class. Please let me know if you have a question or find a mistake.

1. Let $R > 0$ be given. For which choices of constants c_j is

$$\omega = \sum_{j=1}^{n+1} c_j x^j dx^1 \wedge \cdots \wedge dx^{j-1} \wedge dx^{j+1} \wedge \cdots \wedge dx^{n+1}$$

   the Riemannian volume form on the sphere $\partial B_R = \{x \in \mathbb{R}^{n+1} \colon |x| = R\}$, for some choice of orientation?

2. Use the result of the previous exercise and Stokes' theorem to compute the ratio of the volume of ∂B_R to the volume of the ball in $B_R = \{x \in \mathbb{R}^{n+1} \colon |x| \leq R\}$.

3. Exercises 7 and 8 from page 16 of Sadun's notes `https://arxiv.org/pdf/1604.07862.pdf`.

4. Find a basis for $H^k(S^1)$, where $S^1 = \{(x, y) \in \mathbb{R}^2 \colon x^2 + y^2 = 1\}$, for every $k \in \{0, 1, 2, \dots\}$.

   Hint: Fill in the details for the solution sketched in Section 5.1 of Sadun's notes.