MA 595: Semiclassical Analysis in Quantum Mechanics
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Semiclassical analysis is the mathematical study of wave-particle correspondence. The simplest example is a light or sound wave, which at scales much larger than the wavelength (the familiar setting for light and sound) travels along the same straight lines as any point particle. With the advent of quantum mechanics, this correspondence was shown to apply in varying degrees to all matter, with salient features of complicated wave behavior explained in simpler particle terms.

This course will begin with the first general semiclassical approximation, the WKB approximation. Our focus will be the fundamental problem of computing spectra (usually eigenvalues) and the associated states (eigenvectors). Under broad conditions these are governed by universal asymptotics, which go back to work of Rayleigh on acoustics and black body radiation, and have appeared in various guises in Weyl's law for eigenvalues of the Laplacian, in Thomas and Fermi's theory of electronic structure, and in the far-reaching generalizations and refinements these have continued to see.

To understand these, we will develop technical tools, including pseudodifferential operators, functional calculus, and trace formulas. To some extent we will follow the textbooks 'Spectral Asymptotics in the Semi-Classical Limit' by Dimassi and Sjostrand, and 'Semiclassical Analysis' by Zworski, as well as the online course notes 'An Introduction to Semiclassical Analysis' by Nonnenmacher, but our approach will be more elementary, and we will not go as far as those references do.

We will then study recent results on the contribution of boundaries to spectral asymptotics for the Schrodinger equation. Asymptotic methods are important here because this equation becomes too complicated to be solved directly, even numerically, as the number of particles grows. Our asymptotics make it possible to compute energies accurately and efficiently, yielding improvements to approximations used in density functional theory.