Kiril Datchev
MA 595
Fall 2026
Quiz 1

Name:____

1. Let $\psi(r)$ be a (not necessarily normalized) ground state wave function for the hydrogen atom as a function of r, the distance to the proton. Let a be the Bohr radius.
   (a) Write a formula for $\psi(r)$.
   (b) Write a formula, involving one or more integrals which you need not evaluate or simplify, for the probability that the electron is within one Bohr radius of the proton.

(over)

2. Let H be a linear operator, and let u_1 and u_2 be solutions to $i\partial_t u_j = Hu_j$, with $u_j|_{t=0} = \psi_j$, $H\psi_j = E_j\psi_j$, for $j = 1, 2$. Suppose $E_1 \neq E_2$. Let $u = (u_1 + u_2)/\sqrt{2}$.

   (a) Find a time $T > 0$ such that $u(T)$ is proportional to $u(0)$.

   (b) Give a condition on E_1 and E_2 which guarantees that there is a time $T > 0$ such that $u(T) = u(0)$.

Solutions:

1. (a) is $\psi(r) = e^{-r/a}$ and (b) is $\int_0^a r^2\psi(r)^2\,dr \Big/ \int_0^\infty r^2\psi(r)^2\,dr$.

2. (a) We have
$$u(T) = (e^{-iTE_1}\psi_1 + e^{-iTE_2}\psi_2)/\sqrt{2}.$$
Equating with $Ku(0)$ gives
$$e^{-iTE_1}\psi_1 + e^{-iTE_2}\psi_2 = K\psi_1 + K\psi_2.$$
Matching coefficients on ψ_1 and ψ_2 gives
$$e^{-iTE_1} = e^{-iTE_2},$$
which means we want $TE_1 - TE_2$ to be an integer multiple of 2π. For example, we may take $T = 2\pi/|E_1 - E_2|$, or more generally $T = 2\pi m/|E_1 - E_2|$ for any positive integer m.
(b) Doing the same thing with $K = 1$ leads to wanting TE_1 and TE_2 to both be integer multiples of 2π. A sufficient condition is $E_1 = 2E_2$, in which case $T = 2\pi/E_2$ works.