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MA 595
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Quiz 10

Name:_____

Find f_0, f_1, and f_2 such that if

$$u = ae^{-\varphi/h}, \qquad (\varphi')^2 = V,$$

then

$$e^{\varphi/h}(h^2u'' + Eu - Vu) = f_0 + hf_1 + h^2f_2.$$

(If you have extra time, think about how to smoothly solve $(\varphi')^2 = V$ near a double root of V.)

Solution:

From

$$u' = (a' - h^{-1}\varphi' a)e^{-\varphi/h}, \qquad e^{\varphi/h}h^2 u'' = h^2 a'' - 2h\varphi' a' - h\varphi'' a + (\varphi')^2,$$

get

$$f_0 = (\varphi')^2 + E - V = E, \qquad f_1 = -2h\varphi' a' - h\varphi'' a, \qquad f_2 = a''.$$