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MA 595
Fall 2026
Quiz 11

Name:_____

Complete each sentence.

1. $r^\ell Y_\ell^m(\theta_1, \theta_2)$ is a ...

   if and only if Y_ℓ^m is a ...

2. The purpose of finding φ, a, and E such that $((\varphi')^2 + E - V)a - h(2a'\varphi' + a\varphi'') + h^2 a''$ is as small as possible is ...

3. Using φ such that $(\varphi')^2 = V$ leads to good results when V is ...

Solutions:

1. harmonic homogeneous polynomial ... spherical harmonic of degree ℓ, i.e. an eigenfunction of the spherical Laplacian with eigenvalue $-\ell(\ell+1)$ (or maybe even $-\ell(\ell+d-2)$).
2. to get a quasimode of the semiclassical Schrödinger operator $-h^2\frac{d^2}{dx^2}+V$ near the energy level E, and hence deduce that there is an energy level nearby (the smaller that quantity, the greater the accuracy to which we know the energy level.)
3. everywhere positive except for a unique zero where the second derivative is nonzero.