

Name: \_\_\_\_\_

Let  $N$  be a given positive integer and let  $n(x) = (2/\pi) \sum_{j=1}^N \sin^2(jx)$ . Recall that  $\int_0^\pi n(x) dx = N$ .

1. What is  $n(x)$  called?
2. Use a symmetry of  $n(x)$  to compute  $\int_0^{\pi/2} n(x) dx$ . Why does  $n(x)$  have this symmetry?
3. Evaluate  $n(\pi/2)$  in the cases  $N$  odd and  $N$  even.

Solutions:

1. The ground state particle density.
2.  $n(x) = n(\pi - x)$  because each eigenstate  $\sin(jx)$  is either even or odd with respect to the line  $x = \pi/2$ . This can be checked using a trigonometric identity, or alternatively it follows from the fact that the operator  $(S\psi)(x) = \psi(\pi - x)$  commutes with the Hamiltonian  $-\frac{1}{2} \frac{d^2}{dx^2}$  on  $[0, \pi]$ . Hence  $\int_0^{\pi/2} n = N/2$ .
3.  $\sin^2(j\pi/2)$  is 1 when  $j$  is odd and 0 when  $j$  is even. Thus if  $N$  is odd then  $(2/\pi) \sum_{j=1}^N \sin^2(jx) = (N+1)/\pi$  and if  $N$  is even then  $(2/\pi) \sum_{j=1}^N \sin^2(jx) = N/\pi$