

Name: _____

Let $V: \mathbb{R} \rightarrow \mathbb{R}$ be continuous, let E be a real number, and let $\psi: \mathbb{R} \rightarrow \mathbb{R}$ solve $-\frac{1}{2}\psi'' + V\psi = E\psi$.

1. Prove that if $\psi \in L^2(\mathbb{R})$ and the whole real line is classically forbidden, then ψ is identically zero. *Hint:* Multiply the equation through by ψ and integrate both sides.
2. Complete the sentence: The Sturm Comparison principle states that if $\psi_2: \mathbb{R} \rightarrow \mathbb{R}$ solves $-\frac{1}{2}\psi_2'' + V\psi_2 = E_2\psi_2$ with $E_2 > E$, then ...

Solutions:

1. We have $-\frac{1}{2} \int \psi'' \psi + \int V \psi^2 = \int E \psi^2$, or $\frac{1}{2} \int (\psi')^2 = \int (V - E) \psi^2$. The left side is ≥ 0 , and if $V > E$ everywhere then the right side is ≤ 0 , which means both sides are zero, which means ψ is identically zero.
2. between any pair of successive zeroes of ψ lies a zero of ψ_2 .