

Name: \_\_\_\_\_

1. Let  $H$  be a self-adjoint Hamiltonian, and let  $u$  be a vector in Hilbert space, and let  $f$  be a bounded continuous function. Write the formula for  $\langle u, f(H)u \rangle$  in terms of the spectral projection  $\Pi(E)$ . (Recall that  $\Pi(E)$  is the projection onto states of energy at most  $E$ , i.e. the projection onto the part of the spectrum  $\leq E$ .)
2. Choose a specific Hamiltonian we have studied in class  $H$ , a vector  $u$ , and an energy  $E$ , and write out the spectral projection  $\Pi(E)u$  explicitly.

Solutions:

1.  $\langle u, f(H)u \rangle = \int_{-\infty}^{\infty} f(E) d\langle u, \Pi(E)u \rangle.$

2.  $H = -\frac{1}{2} \frac{d^2}{dx^2}$  for particle in a box  $[0, \pi]$ ,  $u(x) = \sin(x)$ ,  $E = 0.7$ ,  $\Pi(E)u = u.$