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Quiz 6

Name:_____

Let $E$ be a real number, $h > 0$, and $V$ a twice continuously differentiable function on an interval $(A_1, A_2)$ such that $V(x) > E$. We constructed solutions to $-h^2\psi'' + V\psi = E\psi$ of the form

$$\psi(x) = a(x)e^{\varphi(x)/h}(1 + r(\varphi(x))),$$

where $r(\varphi(x))$ is a small remainder, under a suitable technical assumption.

1. What are the formulas for $a$ and $\varphi$ in terms of $V$ and $E$?
2. Find a $V$ such that $r(\varphi(x)) = 0$ for all $x$ in $(A_1, A_2)$. If you can, find all such $V$.

Solutions:

1. $a(x) = (V(x) - E)^{-1/4}$, $\varphi(x) = \int_{x_0}^{x} \sqrt{V - E}$, where $x_0$ is any point in the interval.
2. We need $a'' = 0$, which corresponds to $(V(x) - E)^{-1/4} = Ax + B$, or $V(x) = E + (Ax + B)^{-1/4}$ for some constants $A$ and $B$. If $A = 0$ we get the simple case that $V$ is a constant greater than $E$ on $(A_1, A_2)$.