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Quiz 8

Name:_____

1. Let H be a self-adjoint operator, and suppose

$$\|(H-\lambda)u\| \leq \varepsilon \|u\| \neq 0. \tag{1}$$

   What can we conclude about the distance from λ to the spectrum of H?

2. Let

$$H = \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix}, \qquad \lambda = 0.$$

   Find

$$u = \begin{pmatrix} x \\ y \end{pmatrix}$$

   such that (1) is satisfied with ε as small as possible.

Solutions:

1. The distance is at most ε.
2. We know $\varepsilon = 1$ is because the spectrum is $\{-3, 1\}$. We have $\|(H - \lambda)u\|^2 = x^2 + 9y^2$ and $\|u\|^2 = x^2 + y^2$, and these are equal if and only if $y = 0$. So we may take

$$u = \begin{pmatrix} x \\ 0 \end{pmatrix}$$

with any choice of $x \neq 0$.