Kiril Datchev
MA 595
Fall 2026
Quiz 9

Name:_____

1. List five linearly independent harmonic homogeneous polynomials in $\mathbb{R}^2$, as simple as possible.
2. Let f and g be functions in $L^2(\mathbb{R})$. If $g(x) = e^{-x^2}$, and $f \otimes g = g \otimes f$, then what can we conclude about f?

Solutions:

1. The simplest choices are 1, x, y, $x^2 - y^2$, xy, keeping the degree as low as possible.
2. $f(x)e^{-y^2} = e^{-x^2}f(y)$. Cross multiplying gives $e^{x^2}f(x) = e^{y^2}f(y)$. Each side must be a constant, so f is a constant multiple of g.