Modeling Deformable Cells Using Spherical Harmonics

SURF SUMMER UNDERGRADUATE RESEARCH FELLOWSHIP

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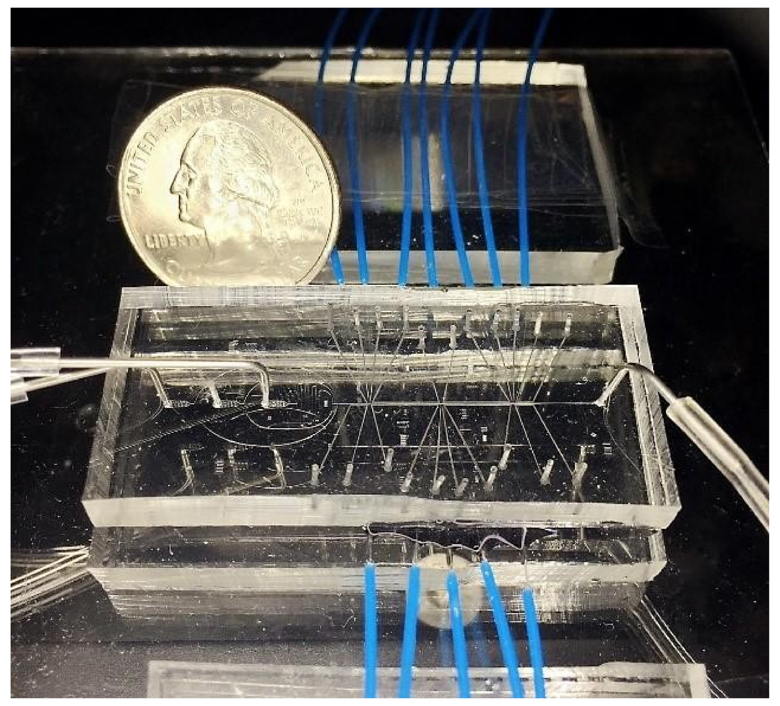
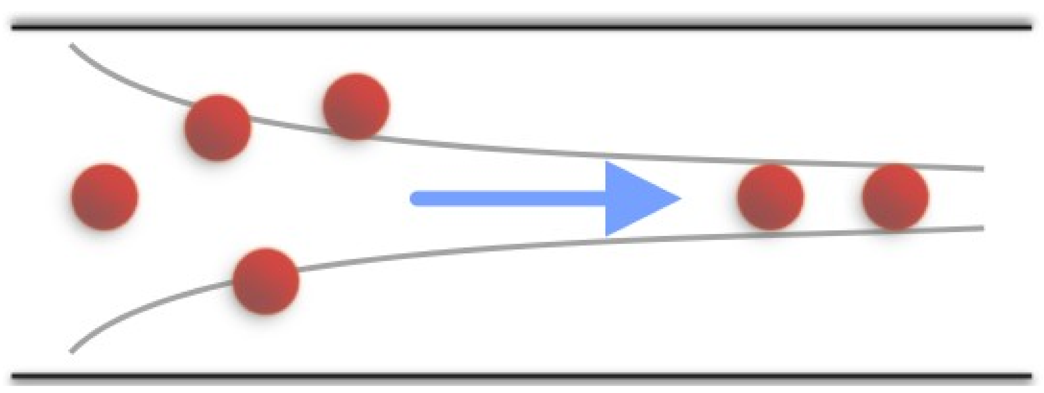
Background

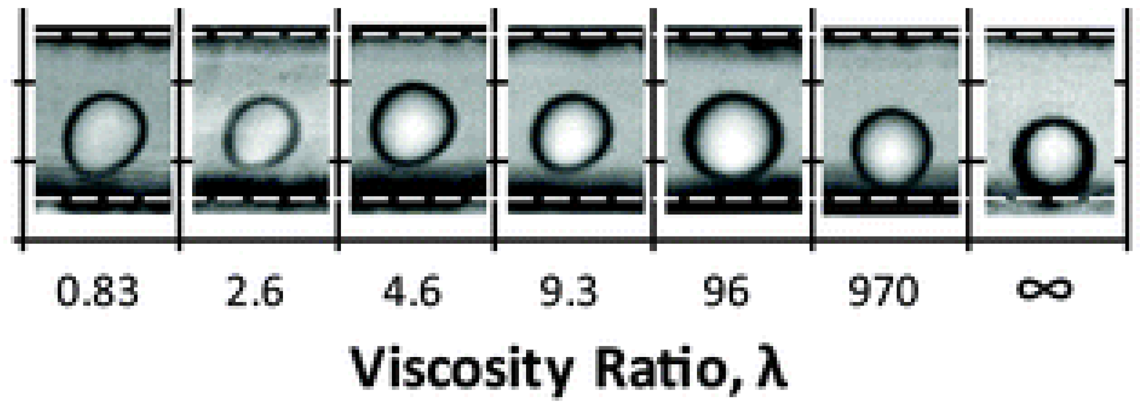
- Flow cytometry is used in various biological applications for categorizing different cells, like detecting rare diseases in blood tests. They are designed to exploit the inertial focusing of rigid particles.



- Inertial focusing is the effect of particles to move towards an equilibrium position in the channel based on the type of flow.

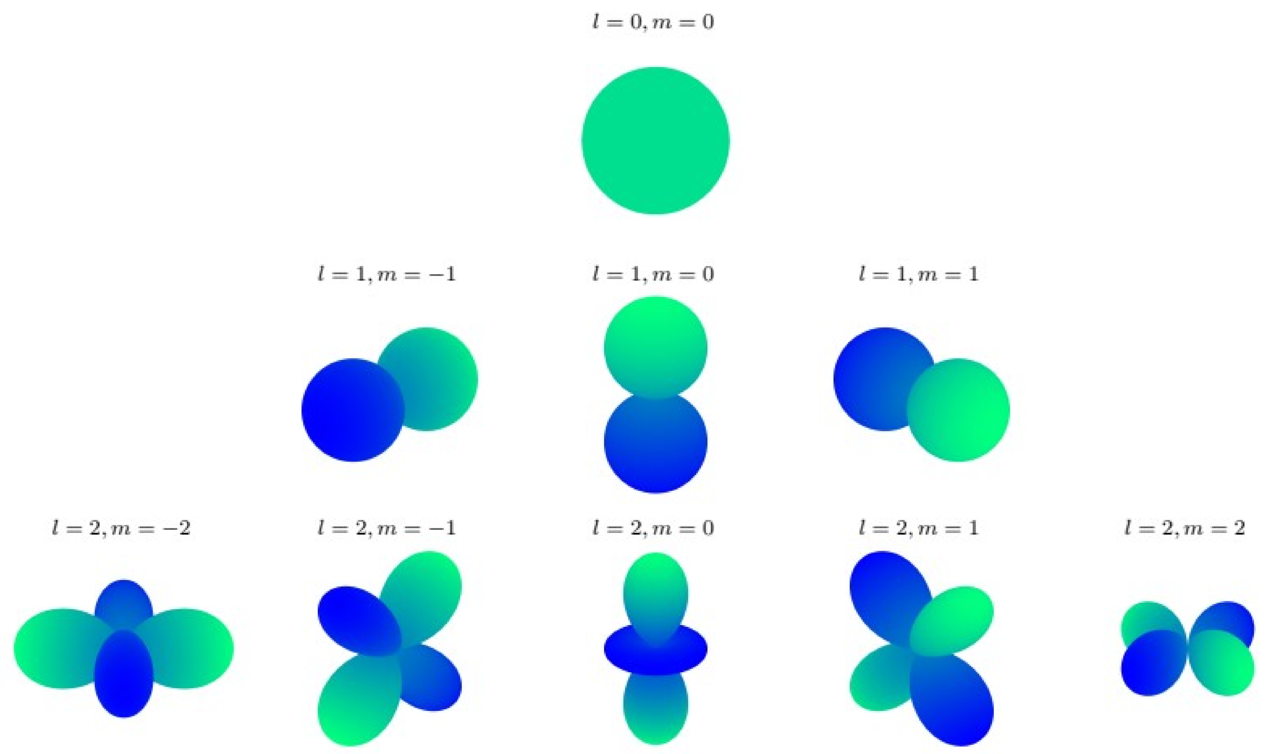


- However, experiments show that the inertial focusing of deformable cells doesn't agree with models for rigid particles, indicating a dependence on the shape of the particle during flow.



- How can we describe the shape of cells during flow?

Spherical Harmonics



- Spherical harmonics provide an orthonormal basis on the sphere, and provide a natural way to describe spherical shapes. We can express the cell shape as a linear combination of these functions.
- We use up to the *l=2* band in our approximation.

Volume Constraints

- We constrain the coefficients by requiring a constant volume of the spherical shape. The resulting volume integral then involves combinations of the spherical harmonics

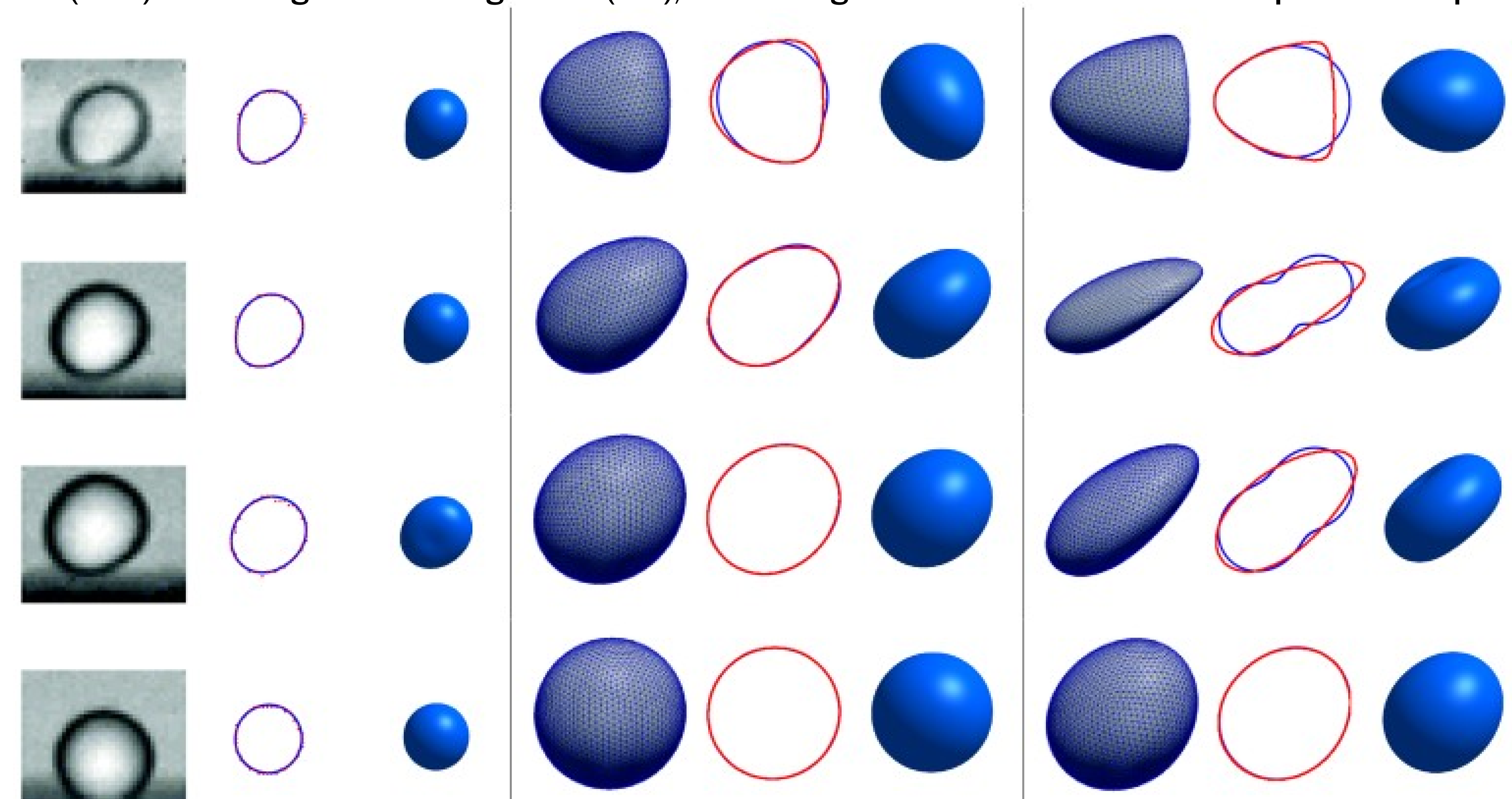
$$\mathrm{Vol}(f) = \frac{1}{3}\int_0^{2\pi}\int_0^{\pi}\left(\sum_{l,m}\alpha_{l,m}Y_{l,m}\right)^3 \sin(\phi)\,d\phi\,d\theta$$

- Solving the equation for $\alpha_{0,0}$ results in a depressed cubic which have known solutions. Below is the volume constraint for *l=2*.

$$\begin{aligned}\mathrm{Vol}(f) = {} & \frac{\boldsymbol{\alpha}_{0,0}^3}{6\sqrt{\pi}} + \frac{\boldsymbol{\alpha}_{0,0}}{2\sqrt{\pi}}\cdot\Big(\alpha_{1,0}^2 + \alpha_{1,1}^2 + \alpha_{1,-1}^2 + \alpha_{2,1}^2 + \alpha_{2,2}^2 + \alpha_{2,-1}^2 + \\ & \alpha_{2,2}^2 + \alpha_{2,0}^2\Big) + \frac{1}{42\sqrt{5\pi}}\cdot\Big(5\alpha_{2,0}(2\alpha_{2,0}^2 + 3\alpha_{2,1}^2 + 3\alpha_{2,-1}^2 - 6\alpha_{2,2}^2 - 6\alpha_{2,-2}^2) \\ & +42\sqrt{3}\alpha_{1,0}(\alpha_{1,1}\alpha_{2,1} + \alpha_{1,-1}\alpha_{2,-1}) - 21\alpha_{1,1}^2(\alpha_{2,0} - \sqrt{3}\alpha_{2,2}) + +42\alpha_{1,0}^2\alpha_{2,0} \\ & 15\sqrt{3}\alpha_{2,2}(\alpha_{2,1}^2 - \alpha_{2,-1}^2) + 42\sqrt{3}\alpha_{1,1}\alpha_{1,-1}\alpha_{2,2} - 21\alpha_{1,-1}^2(\alpha_{2,0} + \sqrt{3}\alpha_{2,2}) \\ & +30\sqrt{3}\alpha_{2,1}\alpha_{2,-1}\alpha_{2,2}\Big)\end{aligned}$$

Results

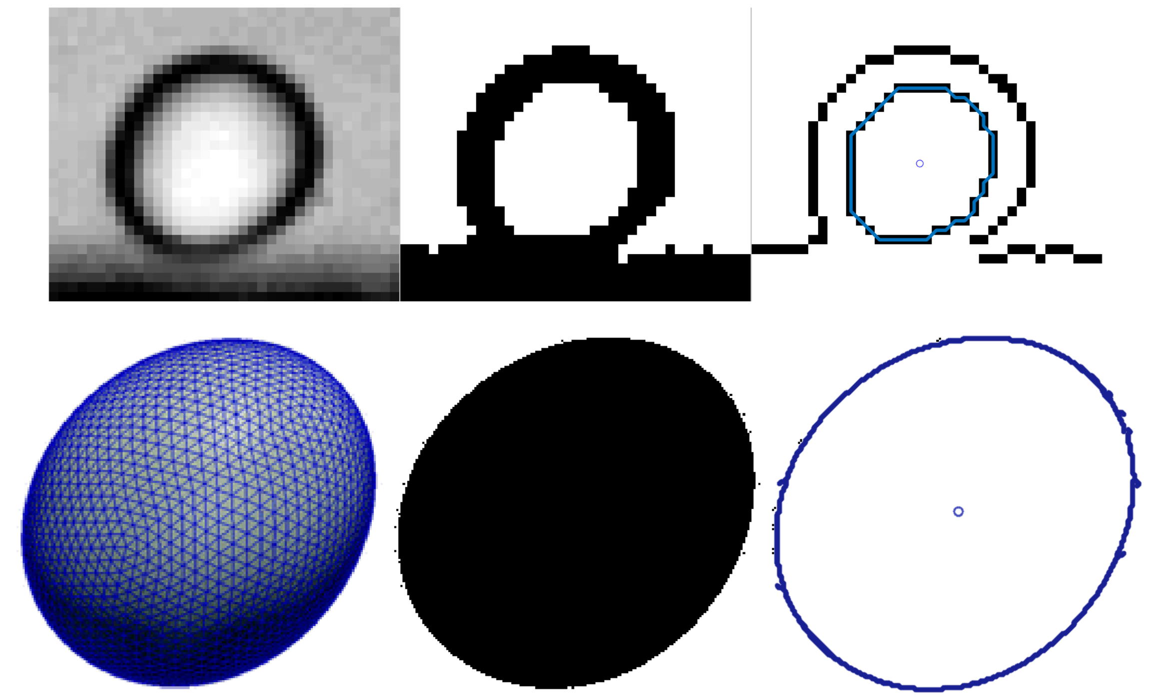
In each dataset the left is the original image, the middle shows the fitting of the spherical harmonic cross section (blue) to the edge of the image data (red), and the right is the final reconstructed spherical shape.



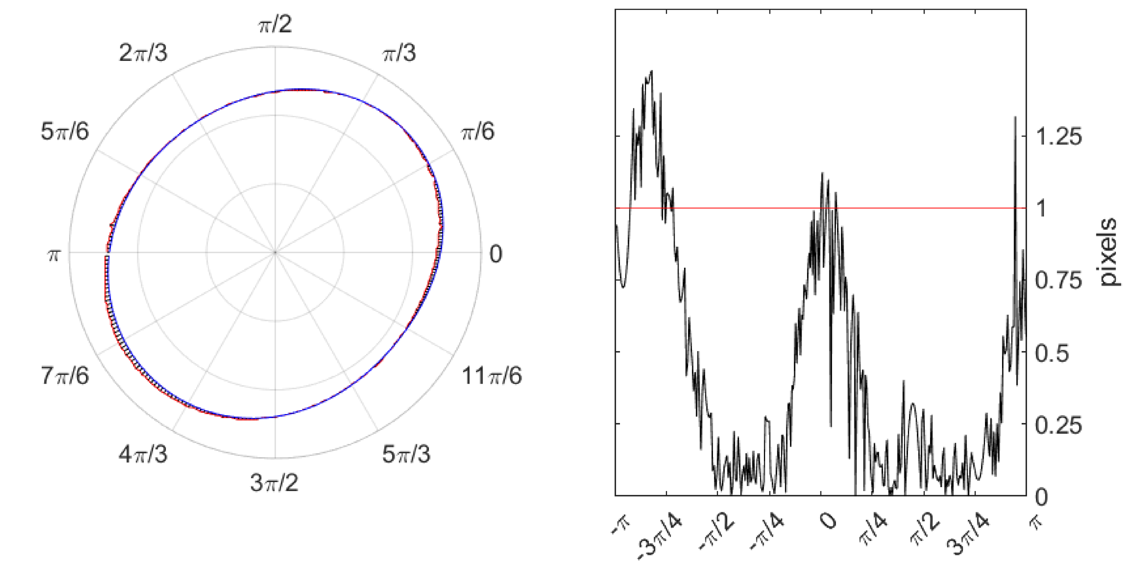
Hur, et al. | Schaaf and Stark (*Re=10*) | Schaaf and Stark (*Re=100*)

Code

- We gather 2D image data from previous simulations and experiments.
- We perform edge detection on the images to extract the edge of the cell.



- We then frame this as an optimization problem and try to minimize the area between the edge and the cross section of spherical harmonics.



Future Work

- Calculating lift and drag forces on each of the shapes expressed as spherical harmonics to determine the equilibrium positions

References

- *Gregory A Cooksey | NIST*. (2022b, December 8). NIST. https://www.nist.gov/people/gregory-cooksey
- S. C. Hur, N. K. Henderson-MacLennan, E. R. McCabe, and D. Di Carlo, Deformability-based cell classification and enrichment using inertial microfluidics, Lab on a Chip 11, 912 (2011).
- C. Schaaf and H. Stark, Inertial migration and axial control of deformable capsules, Soft matter 13, 3544 (2017).