☆ Mean Value Theorem & What Derivatives Tell Us

Warm Up → Announcements

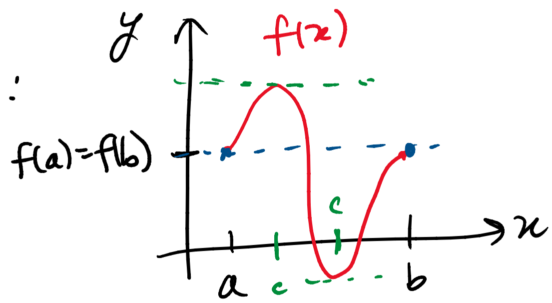
Rolle's Theorem:

If $f(x)$ is:

1) continuous on $[a,b]$
2) differentiable (a,b)
3) $f(a) = f(b)$

Then there is at least one value of x, call it c, where $a < c < b$ and $f'(c) = 0$

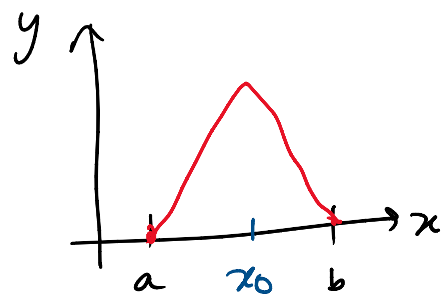
Ex:



1) $f(x)$ is continuous on $[a,b]$ ✓
2) $f(x)$ is diff on (a,b) ✓
3) $f(a) = f(b)$ ✓

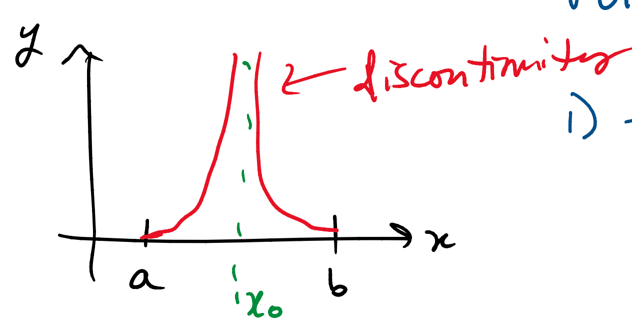
Then there exists a c such that $f'(c) = 0$

Counter Examples:



1) $f(x)$ is continuous ✓
2) $f(x)$ is not diff at at x_0 X

Rolle's Thm does not apply



1) $f(x)$ continuous? X

Rolle's Thm does not apply.

Ex: $f(x) = x^3 - 4x + 6$ $\quad [0,2]$

Ex: $f(x) = x^3 - 4x + 6$
Verify Rolle's Theorem on $[0,2]$

1) Is $f(x)$ continuous on $[0,2]$? ✓ polynomial are continuous on $(-\infty,\infty)$
2) Is $f(x)$ diff'ble on $(0,2)$? ✓ are diff'ble on $(-\infty,\infty)$
3) Check $f(0) \stackrel{?}{=} f(2)$ ✓

$f(0) = 0^3 - 4\cdot 0 + 6 = 6$
$f(2) = 2^3 - 4\cdot 2 + 6 = 8 - 8 + 6 = 6$

Rolle's Thm says there exists a c where $0 < c < 2$ and $f'(c) = 0$

NOTE: Rolle's Thm doesn't tell us the value of c.

To find c: Solve $f'(x) = 0$, want $0 < x < 2$

$f'(x) = 3x^2 - 4 = 0$
$3x^2 = 4$
$\sqrt{x^2} = \sqrt{\frac{4}{3}}$
$x = \frac{\pm 2}{\sqrt{3}}$

want $0 < x < 2$

$\boxed{x = \frac{2}{\sqrt{3}}}$

Q: For what functions $f(x)$ on $[a,b]$ is there a point where the tangent line is parallel to the secant line?

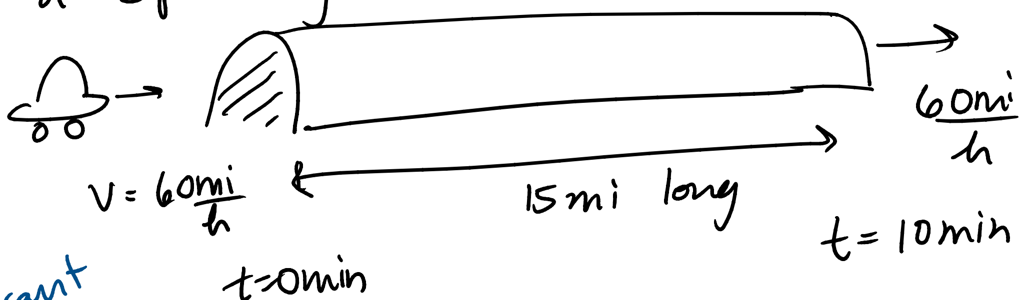
A: Mean Value Theorem:

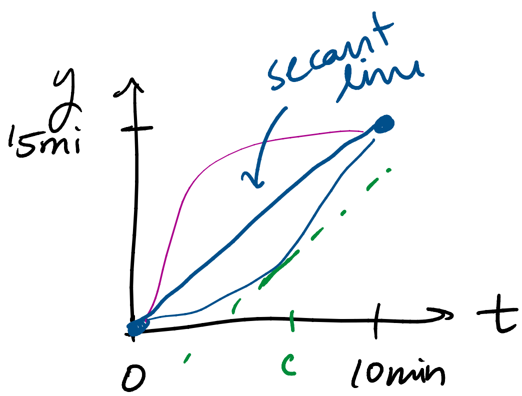
If $f(x)$ is:

1) continuous on $[a,b]$
2) differentiable on (a,b)

Then there is at least one location c where $a<c<b$ and $\underbrace{f'(c)}_{\text{slope of the tangent line}} = \underbrace{\dfrac{f(b)-f(a)}{b-a}}_{\text{slope of secant line}}$

Ex: Catch a speeding car





Let $s(t)$ be the position of the car

slope of secant line $\dfrac{s(10)-s(0)}{(10-0)\text{ min}} = \dfrac{15\text{mi}}{10\text{min}}$

$= \dfrac{15\text{mi}}{\frac{10}{60}\text{hr}} = 90 \text{ mi/hr}$

MVT: (Mean Value Theorem) $s'(c) = 90\frac{mi}{h}$

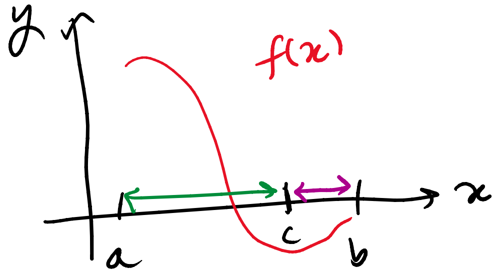
car must be going 90 mi/hr at some point in the tunnel

(Mean Value Theorem) can ... at some point in the ...

4.3 What the Derivative Tells Us (Part 1)

Before:

Given $f(x)$ ⟶ Find $f'(x)$

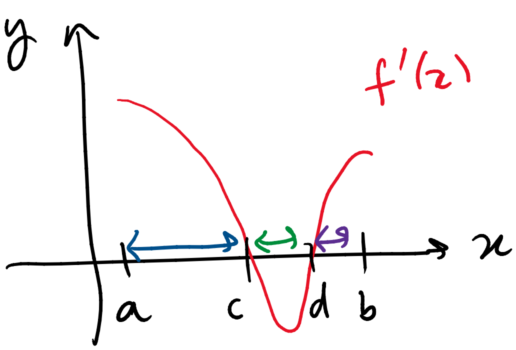


on (a,c) $f(x)$ is decreasing
$f'(x) < 0$

(c,b) $f(x)$ is increasing
$f'(x) > 0$

TODAY

Given $f'(x)$

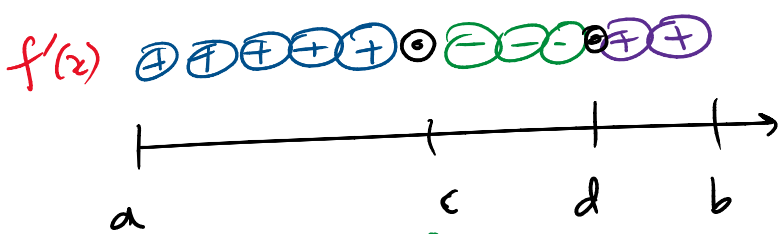


(a,c) $f'(x) > 0$
$f(x)$ increasing

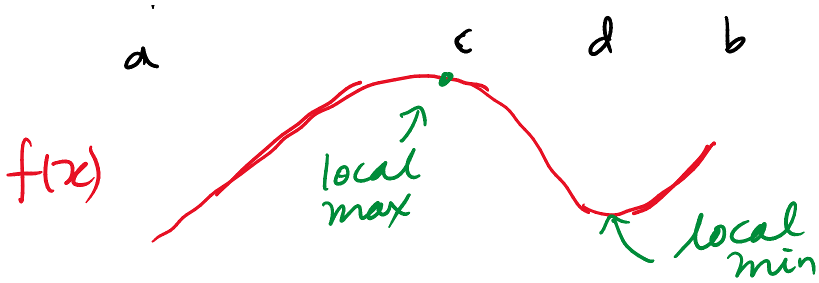
(c,d) $f'(x) < 0$
$f(x)$ decreasing

(d,b) $f'(x) > 0$
$f(x)$ increasing

$f'(c)=0$ $f'(d)=0$ ⟵ Lesson 20 → c, d are critical points possibly be a local min or local max



Number line

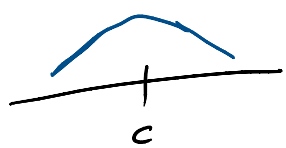
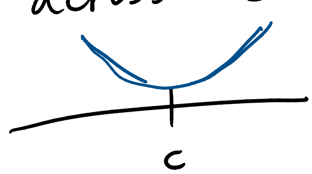
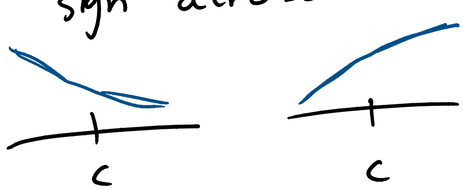


← sketch of f(x)

First Derivative Test:

Let $x=c$ be a critical point
(i.e. $f'(c)=0$ or $f'(c)=\text{DNE}$)

Then we have 3 cases:

case	$f'(x)$	c
(i)	$f'(x)$ changes from ⊕ to ⊖ across c 	c is a local max
(ii)	$f'(x)$ changes from ⊖ to ⊕ across c 	c is a local min
(iii)	$f'(x)$ doesn't change sign across c 	c is neither a local max nor a local min

Ex: $f(x) = 3x^5 - 5x^3$

critical points: $f'(x) = 0$

$\cancel{f'(x) = DNE}$ polynomial

$$15x^4 - 15x^2 = 0$$

$$15x^2(x^2 - 1) = 0$$

$$15x^2(x-1)(x+1) = 0$$

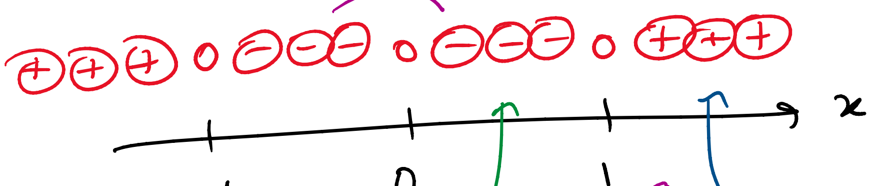
critical points $x = 0, 1, -1$

Draw a number line

neither a local max or min

Check the sign of $f'(x)$

plug in some sample values of x



check $f'(2)$ $15(2)^2(2-1)(2+1) = +$

local max @ $x=-1$

check $f'(0.5)$

$15(0.5)^2(0.5-1)(0.5+1)$

+ + − + = −

local min @ $x=1$