§4.4. Graphing functions.

Graphing guidelines:

1. Domain
2. Symmetry
3. f', f''
4. critical points, IP
5. Interval: Increase/Decrease, CU/CD
6. Ext. values, IP
7. Asymptotes
8. Intercepts
9. Graphing.

eg 1. $f(x) = \frac{x^3}{3} - 400x$ ① Graph $y = f(x)$.

Sol. 1. Domain : $(-\infty, \infty)$

2. Symmetry : $f(-x) = -f(x)$. odd func

Sym. about origin.

3. $f' = x^2 - 400$ ②

$f'' = 2x$ ③

4. • Critical points : $0 \overset{②}{=} f' = x^2 - 400$

$\iff x = \pm 20$

• (Potential) IP : $0 = f'' \overset{③}{=} 2x \iff x = 0$

5. $f' = (x-20)(x+20)$

	$(-\infty, -20)$	-20	$(-20, 20)$	20	$(20, \infty)$
f'	$+$		$-$		$+$

Increasing interval : $(-\infty, -20)$, $(20, \infty)$

Decreasing " : $(-20, 20)$

	$(-\infty, 0)$	0	$(0, \infty)$
f''	$-$		$+$
	CD	IP	CU

6. Ext. values : • $x=-20$, $f''<0$, local max

• $x=20$, $f''>0$, local mini.

(2nd Der Test)

IP : • $x=0$: f change from CD to CU

$x=0$ is IP

7. $\lim_{x\to\infty} f = \lim_{x\to\infty} x\left(\frac{x^2}{3}-400\right) = \infty$

$\lim_{x\to-\infty} f = \lim_{x\to-\infty} x\left(\frac{x^2}{3}-400\right) = -\infty$

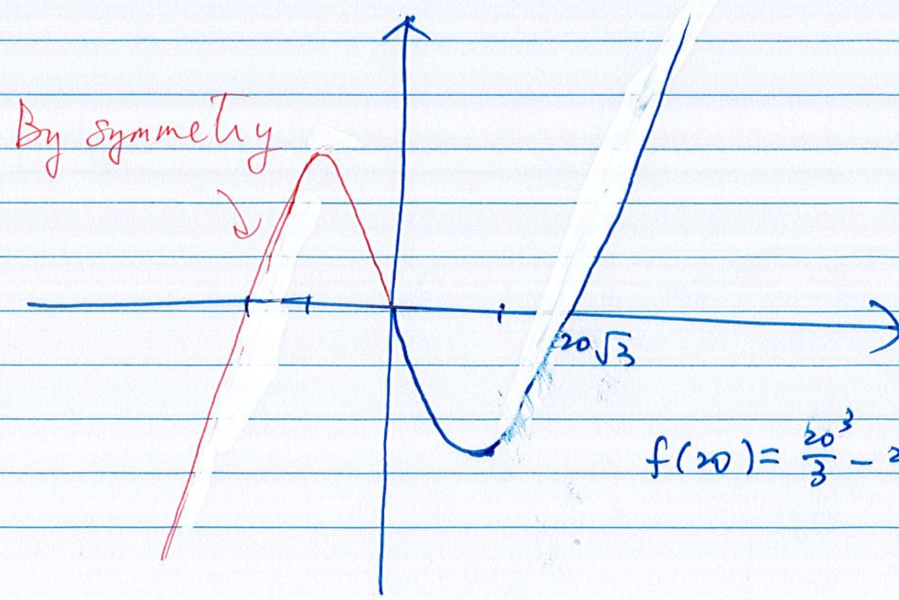
No HA, No VA.

8. Intercepts : $y=0 \iff x\left(\frac{x^2}{3}-400\right)=0$

$\iff x=0$ or $\pm\sqrt{1200}$

x-intercept : 0, $20\sqrt{3}$, $-20\sqrt{3}$

9.



$f(20)=\frac{20^3}{3}-20^3=-\frac{2}{3}\cdot 20^3$

eg 2. Graph $y = f(x) = \dfrac{10x^3}{x^2-1}$ ①

Sol. 1. Domain : $x \neq \pm 1$ $(x^2-1 \neq 0)$

2. Symmetry : $f(-x) = \dfrac{10(-x)^3}{(-x)^2-1} = \dfrac{-10x^3}{x^2-1}$

$= -f(-x)$

odd function

Sym. about origin.

3. Deriv. : $f' = \dfrac{(x^2-1)(30x^2) - 10x^3(2x)}{(x^2-1)^2}$

$= \dfrac{10x^4 - 30x^2}{(x^2-1)^2} = \dfrac{10x^2(x^2-3)}{(x-1)^2}$ ②

$f'' = \dfrac{(x^2-1)^2(40x^3-60x) - (10x^4-30x^2)\,2(x^2-1)(2x)}{(x^2-1)^4}$

$= \dfrac{(x^2-1)\left\{(40x^3-60x)(x^2-1) - (10x^4-30x^2)(4x)\right\}}{(x^2-1)^4}$

$= \dfrac{1}{(x^2-1)^3}\left\{\underbrace{40x^5 - 100x^3 + 60x - (40x^5 - 120x^3)}_{20x^3+60x}\right\}$

$= \dfrac{20x(x^2+1)}{(x^2-1)^3}$ ③

4. Critical points: $0 = f' \iff 10x^2(x^2-3) = 0$

$\iff x = 0$ or $\pm\sqrt{3}$

possible IP: $0 = f'' \underset{3}{\iff} x = 0$

5. Inc/Dec, Concavity:

		$-\sqrt{3}$		0		$\sqrt{3}$	
$f' = \dfrac{10x^2(x-3)(x+3)}{(x-1)^2}$	$+$		$-$		$-$		$+$
	inc		Dec		dec		inc

		-1		0		1	
$f'' = \dfrac{20x(x^2+1)^2}{(x^2-1)^3} = \dfrac{20x(x^2+1)^2}{(x-1)^3(x+1)^3}$	$-$		$+$		$-$		$+$
	CD		CU	IP	CD		CU

6. 2nd Der test $\Rightarrow$ $x = \sqrt{3}$ local min

$x = -\sqrt{3}$ " max

(Inconclusive for $x = 0$)

1st " " $\Rightarrow$ $x = 0$ neither max nor min

IP : $x = 0$

7. Asymp:

HA : $f = \frac{10x^3}{x^2-1} = \frac{10x^3 \cdot \frac{1}{x^2}}{(x^2-1)\frac{1}{x^2}}$

$= \frac{10x}{1-\frac{1}{x^2}}$

$\lim_{x\to\infty} f = \lim_{x\to\infty} \frac{10x}{1-\frac{1}{x^2}}$ $\left(\frac{1}{1-\frac{1}{x^2}} \to \frac{1}{1-0}\right)$

as $x \to \pm\infty$

$= \infty$

$\lim_{x\to-\infty} f = -\infty$

No HA.

UA : $x^2-1 = 0 \iff x = \pm 1$.

$\lim_{x\to(-1)^-} f = \lim_{x\to(-1)^-} \frac{10x^3}{(x-1)(x+1)} = -\infty$ (4)

$\lim_{x\to(-1)^+} f = \lim_{x\to(-1)^+} \frac{10x^3}{(x-1)(x+1)} = \infty$ (5)

(By sym at 0 $\lim_{x\to1^+} f = \infty$

$\lim_{x\to1^-} f = -\infty$)

Slant Asy: (SA)

$$\begin{array}{r} x \\ x^2+0-1 \overline{) x^3+0x^2+0x+0} \\ \underline{x^3+0-x} \\ x \end{array}$$

$$\therefore \frac{x^3}{x^2-1} = x + \frac{x}{x^2-1} \Rightarrow \frac{10x^3}{x^2-1} = 10x + \frac{10x}{x^2-1}$$

$$\Rightarrow \lim_{\substack{x\to\infty \\ (-\infty)}} \left(\frac{10x^3}{x^2-1} - 10x\right) = \lim_{\substack{x\to\infty \\ (-\infty)}} \frac{10x}{x^2-1} = 0$$

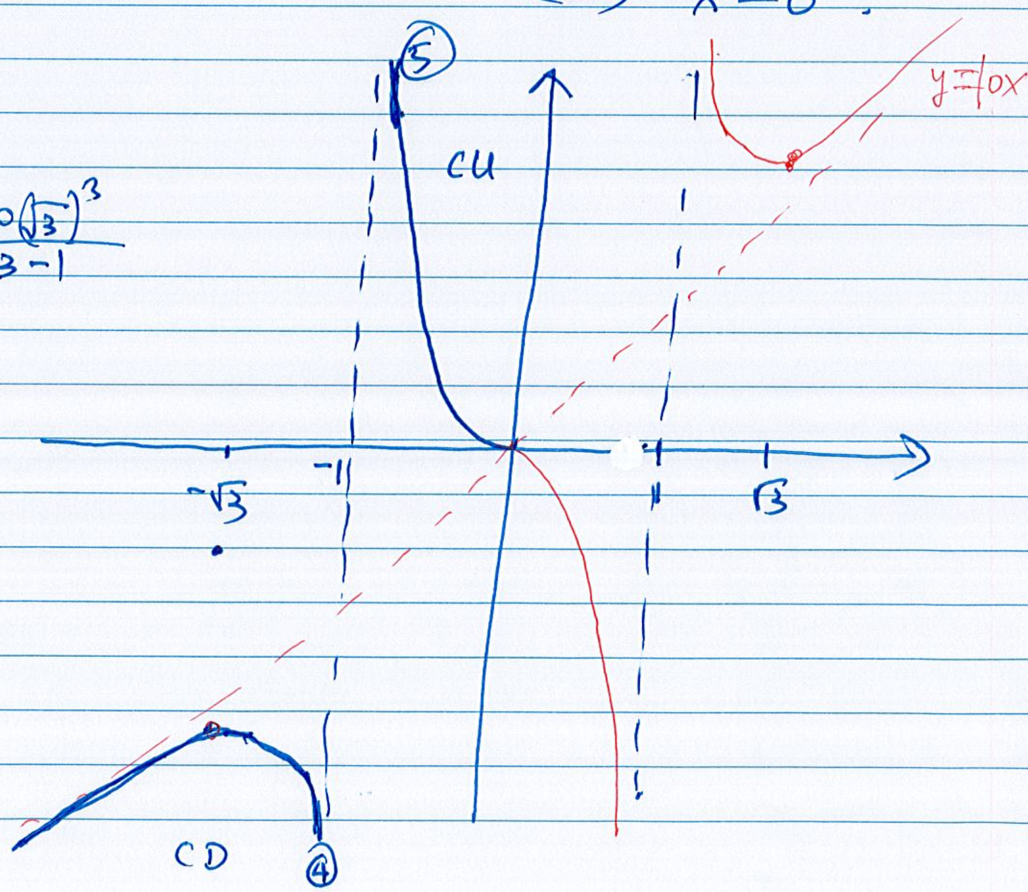
$\therefore$ $y=10x$ is SA

8. Intercepts : $y=0 \iff f(x)=0$

$\iff x=0$.

9.

$$f(-\sqrt{3}) = \frac{-10(\sqrt{3})^3}{3-1}$$



eg 3. Which of the following has slant asymp

a). $y = \dfrac{x^2}{x-1}$

b) $y = \dfrac{x^3}{x^3-1}$

c) $y = \dfrac{x^5}{x^4+2x}$

Ans : Slant asym for rational function occurs when

deg(denominator) = deg(Num) + 1

$\therefore$ (a) and (c) have slant asym.