☆ Graphing Functions - Part 2

Warm Up → Announcements

Q: How can we sketch the graph $f(x)$ without technology?

A: Calculus

Ex: Sketch the graph of $f(x) = e^{-x^2}$

① What is the domain? $(-\infty, \infty)$

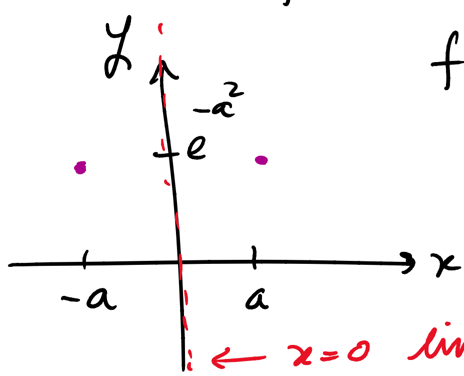
② Is there any symmetry?

check $x = a$ and $x = -a$

$f(a) = e^{-a^2}$ $\qquad f(-a) = e^{-(-a)^2} = e^{-a^2}$

$f(-a) = f(a)$ even function

mirror symmetry across $x = 0$



← $x=0$ line of symmetry

② Find the 1st + 2nd Derivatives

warm up:

$f(x) = e^{-x^2}$

$f'(x) = -2x\,e^{-x^2}$

$f''(x) = [4x^2 - 2]e^{-x^2}$

③ Find the critical points + inflection points

③ Find the $\underbrace{\text{critical points}}$ + inflection points

warm up

$f'(x) = 0$ and $f'(x)$ DNE

$-2x\cancel{e^{-x^2}} = 0$

$\cancel{-2xe^{-x^2}\ \text{DNE?}}$

no

$-2x = 0$

$\boxed{x = 0 \text{ critical point}}$

$f''(x) = 0$

$\boxed{x = \frac{1}{\sqrt{2}} \text{ and } \frac{-1}{\sqrt{2}} \text{ inflection points}}$

④ Chart $f'(x)$ and $f''(x)$

$f'(x)$: + + + + + + 0 − − − − − −

$x = 0$

check $x = -1$

$f'(-1) = -2(-1)e^{-(-1)^2}$

$= 2e^{-1} = \frac{2}{e}$

check $x = 1$

$f'(1) = -2 \cdot 1 \cdot e^{-(1)^2} = -2e^{-1} = \frac{-2}{e}$

−

First Derivative Test

$x = 0$ local max

f'': + + + + 0 − − − − 0 + + + +

++   − −   + +

concave up $\frac{-1}{\sqrt{2}}$ concave down $\frac{1}{\sqrt{2}}$ concave up

⑤ Find the intercepts of $f(x)$

y intercept: $y = f(0) = e^{-0^2} = 1$

function goes through $(0,1)$

x intercept(s): find x such that $y = f(x) = 0$

x-intercept(s): find x such that $y = f(x) = 0$

$\ln(e^{-x^2} = 0)$

$-x^2 = \cancel{\ln(0)}$ undef

no x-intercepts

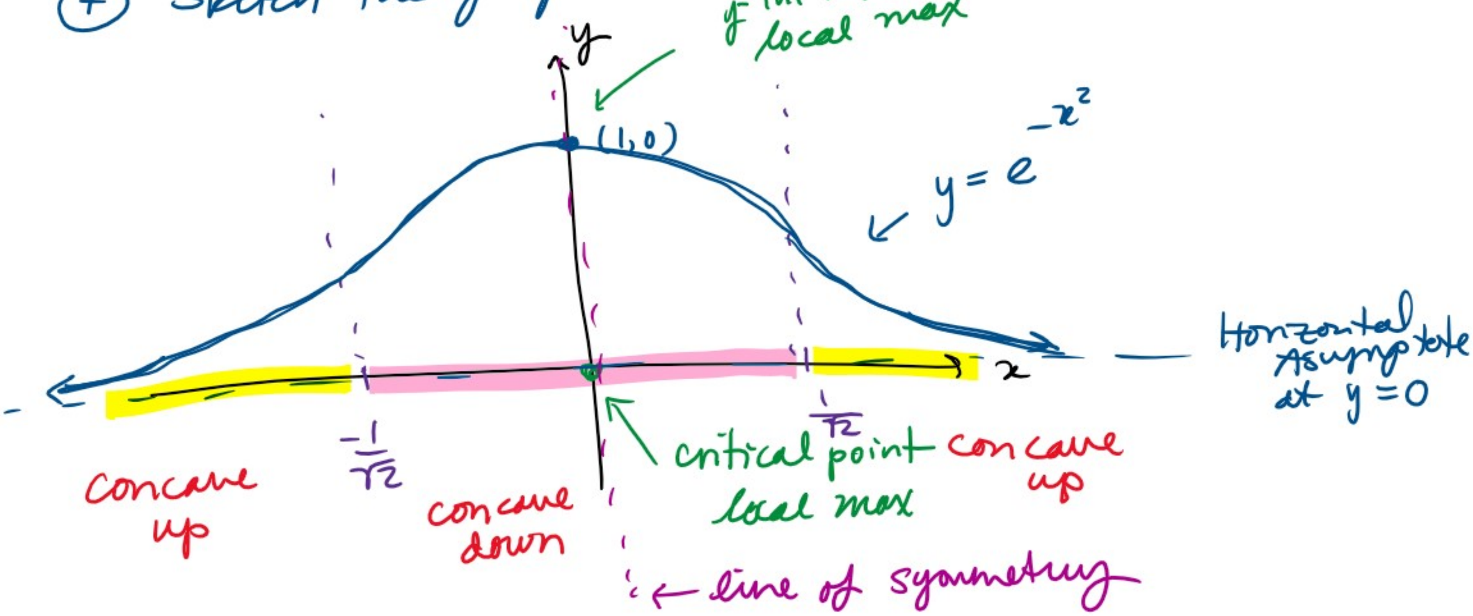
f(x) never crosses the x-axis

⑥ Identify all asymptotes

Horizontal Asymptote at $x=0$

No Vert. Asymp.

⑦ Sketch the graph



NOTE: f'(x) alone is enough to sketch the shape of f(x)

Ex: Sketch f(x) if we know that

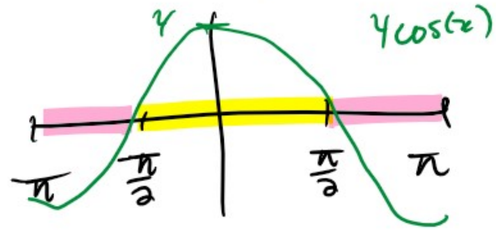
$f'(x) = 4\cos(x)$ on $[-\pi, \pi]$

① Find the critical points + chart $f'(x)$

① Find the critical points & chart $f'(x)$

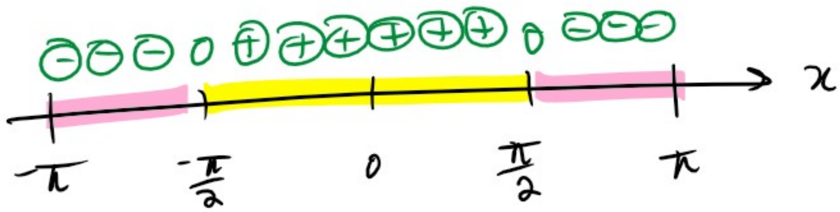
$$f'(x) = 0$$

$$4\cos(x) = 0$$



$$\boxed{\text{Critical points} \quad x = -\frac{\pi}{2}, \frac{\pi}{2}}$$

$f'(x)$



local min

local max

First Deriv Test

Ex: Sketch the graph of $y = f(x) = x^x$

First Derivative Test $\ln\left(y = x^x\right)$

Log Diff

$$\frac{d}{dx}\left(\ln y = \ln(x^x) = x\ln(x)\right)$$

$$\frac{1}{y}\frac{dy}{dx} = 1\cdot\ln(x) + x\cdot\left(\frac{1}{x}\right)$$

$$= \ln(x) + 1$$

$$\frac{dy}{dx} = y\left(\ln(x)+1\right)$$

$$f'(x) = \frac{dy}{dx} = x^x\left(\ln(x)+1\right)$$

critical points:

(Question assumes $x>0$)

$$f'(x)=0$$

$$x^x(\ln(x)+1)=0$$

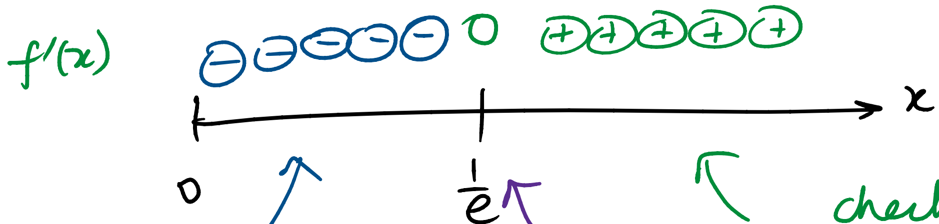
$\cancel{x^x=0}$ assume $x>0$

$$\ln(x)+1=0$$

$$e^{\ln(x)}=e^{-1}$$

$$\boxed{x=e^{-1}=\frac{1}{e}}$$

Chart $f'(x)$



check $x=1$

$$f'(1)=(1)^1\left[\cancel{\ln(1)}^{\,0}+1\right]$$

$$=1\cdot 1=1 \quad +$$

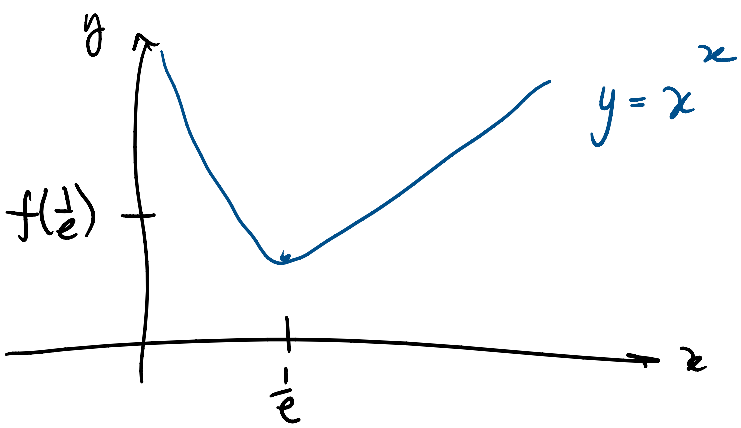
check $x=\frac{1}{e^2}$

$$f'\left(\tfrac{1}{e^2}\right)=\left(\tfrac{1}{e^2}\right)^{\left(\frac{1}{e^2}\right)}\left[\ln\left(\tfrac{1}{e^2}\right)+1\right]$$

$$=+^{+}[-2+1]$$

$$=+^{+}(-1)=-$$

$$\boxed{\begin{array}{l}\text{1st Deriv Test}\\ x=\frac{1}{e}\text{ is a local min}\end{array}}$$



... 2nd Deriv Test to get inflection pts + concavity