Lesson 29 - LEC 100 - 11/7/2022

Antiderivatives

Warm Up → Announcements

Antiderivatives:

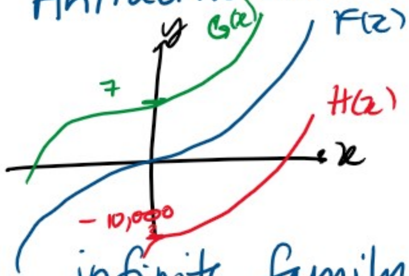
GOAL: undoing the derivative

If $F(x)$ is such that $F'(x) = f(x)$, then $F(x)$ is an antiderivative of $f(x)$

Ex: From warm up: Given $f(x) = 3x^2$

Antiderivatives:

$$\begin{cases} F(x) = x^3 \\ G(x) = x^3 + 7 \\ H(x) = x^3 - 10{,}000 \end{cases}$$



infinite family of antiderivatives

In general, we can write this family of antiderivatives: $F(x) = x^3 + C$ ← represents a vertical shift in $y = F(x)$

Where C is an arbitrary constant

We call C the constant of integration

NOTE: In general, antiderivatives are harder to find than derivatives

→ derivatives: follow the rules

→ derivatives: follow the rules

→ antiderivatives: rules + memorization + creativity

Memorization:

differentiate →

$F(x)$	$F'(x) = f(x)$
x^n	nx^{n-1}
$\cos(x)$	$-\sin(x)$
$\sin(x)$	$\cos(x)$
$\tan(x)$	$\sec^2(x)$
e^x	e^x
$\ln\lvert x\rvert$	$\frac{1}{x}$
$\tan^{-1}(x)$	$\frac{1}{x^2+1}$

← Antiderivatives + C

Ex: Find the antiderivative of $f(x) = -2\sin(x)$

Look at table $F(x) = 2(\cos(x)) + C$

Deriv Rule $\frac{d}{dx}[2g(x)] = 2\frac{d}{dx}[g(x)] = 2g'(x)$

$G(x) = 2g(x) + C$

$$\boxed{F(x) = 2\cos(x) + C}$$

Ex: Find the antiderivative of

$f(x) = 3x^4 + 10 \ldots = 3x^4 + 10x^{-2}$

Ex: Find the antiderivative of

$$f(x) = 3x^4 + \frac{10}{x^2} = 3x^4 + 10x^{-2}$$

antiderivative:

$$F(x) = 3\,\frac{x^{4+1}}{4+1} + 10\,\frac{x^{-2+1}}{-2+1} + C$$

$$= \frac{3x^5}{5} + 10\,\frac{x^{-1}}{-1} + C$$

$$\boxed{F(x) = \frac{3x^5}{5} - \frac{10}{x} + C}$$

Notation: The <u>indefinite integral</u> is used to find the antiderivative of $f(x)$

$$\underbrace{\int f(x)}_{\text{Find the antiderivative of } f(x)} \underbrace{dx}_{\text{This tells us to find the antiderivative with respect to } x} = F(x) + C$$

x is the independent variable

Ex: $$\int \left(3x^4 + \frac{10}{x^2}\right) dx = \frac{3}{5}x^5 - \frac{10}{x} + C$$

Ex: $$\int \left(\frac{2}{\sqrt{x}} + e^{x}\right) dx$$

sum property

$$\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{df}{dx} + \frac{dg}{dx}$$

$$= \int \left(\frac{2}{\sqrt{x}} + e^x\right) dx$$

$$= \int \frac{2}{\sqrt{x}}\,dx + \int e^x\,dx$$

$\int k f(x)\,dx = k \int f(x)\,dx$

$$= \int 2x^{-1/2}\,dx + \int e^x\,dx$$

$$= 2\int x^{-1/2}\,dx + \int e^x\,dx$$

$$\star = 2\left(\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right) + \left(e^x\right) + \underline{C}$$

only need one constant of integration

$$= 2\left(\frac{x^{1/2}}{1/2}\right) + e^x + C$$

$$= \boxed{4x^{1/2} + e^x + C}$$

could have been $2x^{1/2} + C_1 + e^x + C_2$

$= 2x^{1/2} + e^x + \underbrace{(C_1 + C_2)}_{= C}$

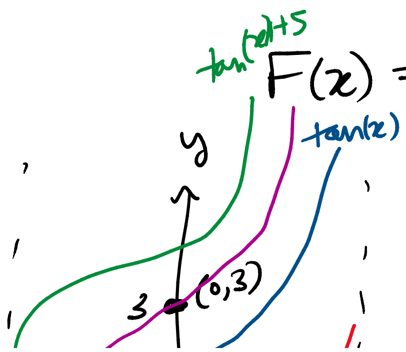
Ex:

$f(x) = \sec^2(x)$

Find $F(x)$ such that $F'(x) = f(x)$ and $F(0) = 3$

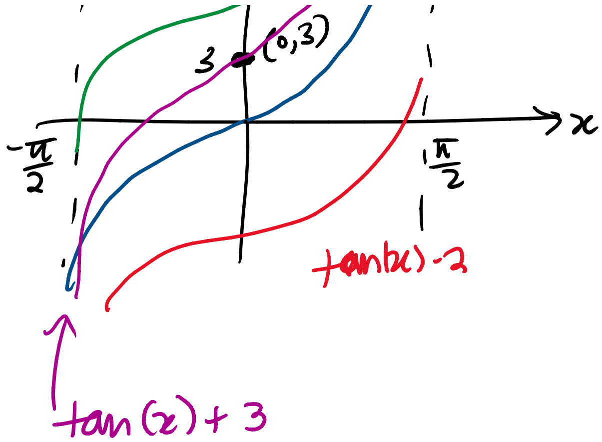
$$F(x) = \int f(x)\,dx = \int \sec^2(x)\,dx$$

$$F(x) = \tan(x) + C$$



want $F(0) = 3$

curve $y = F(x)$ goes thru point $(0,3)$



curve $y = F(x)$ ...
point $(0,3)$

$$F(0) = 3$$
$$[\tan(0) + C] = 3$$
$$0 + C = 3 \rightarrow \boxed{C=3}$$

$$\boxed{F(x) = \tan(x) + 3}$$

We can handle higher order derivatives

<u>Ex:</u> $F''(x) = x$ Find $F(x)$

$$F'(x) = \int F''(x)\,dx = \int x\,dx$$

$$F'(x) = \frac{x^2}{2} + C$$

$$F(x) = \int F'(x)\,dx = \int \left(\frac{x^2}{2} + C\right) dx$$

$$= \frac{1}{2}\left(\frac{x^3}{3}\right) + Cx + D$$ ← second constant of integration

To find C and D, we need 2 points
one for F(x) and another F'(x)

$F(0) = 5$

one for

Given: $F(0) = 5$

$F'(0) = -1$

$F(0) = 5$

$\left[\frac{x^3}{6} + Cx + D\right]\Big|_{x=0} = 5$

$\cancel{\frac{0^3}{6}} + \cancel{C \cdot 0} + D = 5$

$D = 5$

$F'(0) = -1$

$\left[\frac{x^2}{2} + C\right]\Big|_{x=0} = -1$

$\frac{0^2}{2} + C = -1$

$C = -1$

$$\boxed{F(x) = \frac{x^3}{6} - x + 5}$$

$$\begin{cases} F''(x) = x \\ F(0) = 5 \\ F'(0) = -1 \end{cases}$$