Lesson 34 - LEC 100 - 11/28/2022

Substitution Rule - Part 2

Warm Up → Announcements

Think of substitution Rule as "Chain Rule for Integrals"

Warm Up: $\int_0^1 (4x^2+1)^3 x\,dx$

$u = 4x^2+1$

$\frac{du}{8} = x\,dx$

$= \frac{1}{8}\int_1^5 u^3\,du \overset{FTC}{=} \frac{1}{8}\left[\frac{u^4}{4}\right]_1^5$

$= \frac{1}{8}\left[\frac{5^4}{4} - \frac{1^4}{4}\right] = \frac{625-1}{4\cdot 8}$

$= \frac{624}{32} = \boxed{\frac{39}{2}}$

Ex: $\int \sin^4(2x)\cos(2x)\,dx$

$u = \sin^4(2x)$

$du = \underbrace{4\sin^3(2x)}_{\text{too complicated}}\cos(2x)\cdot 2\,dx$

Try $u = \sin(2x)$

$du = \cos(2x)\cdot 2\,dx$

$\frac{du}{2} = \cos(2x)\,dx$

$\int u^4 \frac{du}{2} = \frac{1}{2}\int u^4\,du = \frac{1}{2}\left[\frac{u^5}{5}\right] + C$

$= \boxed{\frac{1}{10}\sin^5(2x) + C}$

Ex: $\int_{\frac{1}{3\pi}}^{\frac{1}{2\pi}} \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \int_{\frac{1}{3\pi}}^{\frac{1}{2\pi}} \sin\left(\frac{1}{x}\right) x^{-2}\,dx$

$u = \frac{1}{x} = x^{-1}$

$du = -x^{-2}\,dx \longrightarrow x^{-2}\,dx = -du$

[illegible] $u = \frac{1}{x} = 3\pi$

$$du = -x^{-2}\,dx \quad \longrightarrow \quad x\,dx = -du$$

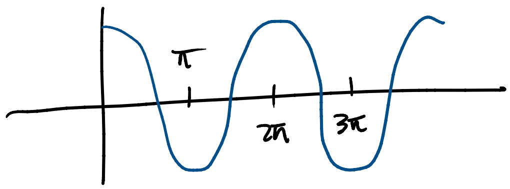
@ $x = \frac{1}{3\pi}$ $\quad u = \frac{1}{x} = 3\pi$

@ $x = \frac{1}{2\pi}$ $\quad u = \frac{1}{x} = 2\pi$

$$\int_{3\pi}^{2\pi} \sin(u)\,(-du)$$

$$\overset{FTC}{=} \Big[\cos(u)\Big]_{3\pi}^{2\pi} = \cos(2\pi) - \cos(3\pi)$$

$$= (1) - (-1)$$



$$= \boxed{2}$$

POLL 3: a) $u = \ln(x)$ $\quad du = \frac{1}{x}\,dx$

$$\int_{e^4}^{e^9} \frac{1}{x\sqrt{\ln(x)}}\,dx = \int_4^9 \frac{du}{\sqrt{u}}$$

@ $x = e^4$ $\quad u = \ln(e^4) = 4$

@ $x = e^9$ $\quad u = \ln(e^9) = 9$

$$= \int_4^9 u^{-1/2}\,du \overset{FTC}{=} \left[\frac{u^{1/2}}{1/2}\right]_4^9 = 2\left[\sqrt{9} - \sqrt{4}\right]$$

$$= 2(3-2) = \boxed{2}$$

b) $u = \sqrt{\ln(x)}$ $\quad du = \frac{1}{2}\left(\ln(x)\right)^{-1/2}\frac{1}{x}\,dx$

$$= \frac{1}{2}\,\frac{1}{x\sqrt{\ln(x)}}\,dx$$

$$2\,du = \frac{1}{x\sqrt{\ln(x)}}\,dx$$

$$\int_{e^4}^{e^9} \frac{1}{x\sqrt{\ln(x)}}\,dx = 2\int_2^3 du$$

@ $x = e^4$ $\quad u = \sqrt{\ln(4)} = 2$

@ $x = e^9$ $\quad u = \sqrt{\ln(9)} = 3$

$$\overset{FTC}{=} 2\Big[u\Big]_2^3 = 2(3-2) = \boxed{2}$$

NOTE: There may be more than one u-substitution that works that works

Q: How do you know when a u-sub works?

A: u-sub should make the integral easier

if after u-sub, the integral is as difficult or harder — try a different u.

Ex: $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} \, dx$

$u = e^x$

$du = e^x \, dx$

$\frac{du}{u} = dx$

$= \int \frac{u - \frac{1}{u}}{u + \frac{1}{u}} \cdot \frac{du}{u}$ ← may work as difficult as original

Notice $\int \frac{\sinh(x)}{\cosh(x)} \, dx$

$u = e^x + e^{-x}$

$du = (e^x - e^{-x}) \, dx$

$\int \frac{du}{u} = \ln(u) + C$

$= \boxed{\ln(e^x + e^{-x}) + C}$