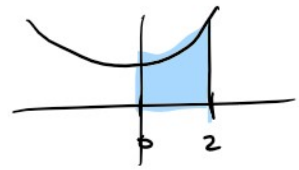
Exam 3 — ver 11

→ Booklets will be returned Tues in Recitation

#1: $f(x) = 4x^2 + 1 \qquad [0, 2]$

$n = 4$ left Rieman sum



$\Delta x = \frac{1}{2}$

$$L_4 = f(0)\Delta x + f(\tfrac{1}{2})\Delta x + f(1)\Delta x + f(\tfrac{3}{2})\Delta x$$

$$= [4\cdot 0^2 + 1](\tfrac{1}{2}) + [4(\tfrac{1}{2})^2 + 1]\cdot\tfrac{1}{2} + [4\cdot 1^2 + 1]\cdot\tfrac{1}{2} + [4\cdot(\tfrac{3}{2})^2 + 1]\cdot\tfrac{1}{2}$$

$$= \frac{1}{2} + \frac{2}{2} + \frac{5}{2} + \frac{10}{2} = \frac{18}{2} = \boxed{9} \qquad \boxed{E}$$

#2: $f(x) = x^2 - 4\cos(x)$ on $[0, 2\pi]$

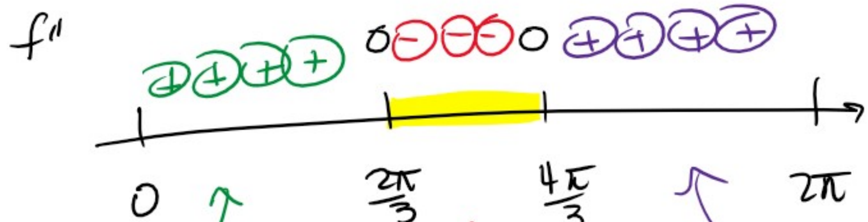
concave downward $\quad f''$ $-$

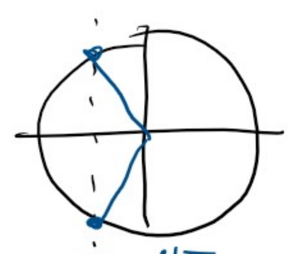
$$f'(x) = 2x - 4(-\sin(x)) = 2x + 4\sin(x)$$

$$f''(x) = 2 + 4\cos(x) < 0$$

$$f''(x) = 0$$
$$2 + 4\cos(x) = 0$$
$$\cos(x) = -\frac{2}{4} = -\frac{1}{2}$$

f''





$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = \frac{2\pi}{3}, \frac{7\pi}{3}$

$f''(0) = 2 + 4\cos(0) = 6$ ⊕

$f''(\pi) = 2 + 4\cos(\pi) = 2 - 4 = -2$ ⊖

$f'(2\pi) = 2 + 4\cos(2\pi)$ ⊕

concave down $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$ $\boxed{B}$

#3: Find c st MVT $f(x) = x + \frac{1}{x}$ on $[1, 3]$

$a < c < b$

MVT: $f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{f(3) - f(1)}{3 - 1}$

$b = 3$

$a = 1$

$$= \frac{\left(3 + \frac{1}{3}\right) - \left(1 + \frac{1}{1}\right)}{2}$$

$$= \frac{\frac{10}{3} - 2}{2} = \frac{\frac{10 - 6}{3}}{2} = \frac{\frac{4}{3}}{2} =$$

$$f'(c) = \frac{2}{3}$$

Solve $f'(x) = \frac{2}{3}$

$$\frac{d}{dx}\left(x + \frac{1}{x}\right) = \frac{2}{3}$$

$$1 - \frac{1}{x^2} = \frac{2}{3}$$

$$\frac{1}{3} = 1 - \frac{2}{3} = \frac{1}{x^2}$$

$x^2 = 3$

$\boxed{x = \sqrt{3}}$ $\boxed{A}$

#4: $\lim_{x \to 0} (1 + 3x)^{1/x}$ → 1^∞ indeterminate

want L'Hopital's $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$$= \ln\left[\lim_{x \to 0} (1 + 3x)^{1/x}\right]$$

$$= e^{\ln\left[\lim_{x\to 0}(1+3x)^{1/x}\right]}$$

$$= e^{\lim_{x\to 0} \ln\left[(1+3x)^{1/x}\right]}$$

$$= e^{\lim_{x\to 0} \frac{\ln(1+3x)}{x}} \longrightarrow \frac{\ln(1)}{0} = \frac{0}{0} \quad \text{L'Hopital's Rule}$$

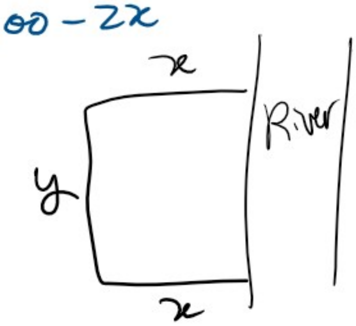
$$\overset{L}{=} e^{\lim_{x\to 0} \frac{\frac{1}{1+3x}\cdot 3}{1}} = e^{\lim_{x\to 0} \frac{3}{1+3x}} = \boxed{e^{3}}$$

$$\boxed{E}$$

#5: $P = 600 = 2x + y \rightarrow y = 600 - 2x$

max $A = xy$

$A = x(600 - 2x) = 600x - 2x^2$



critical points

$A'(x) = 0$

$600 - 4x = 0 \longrightarrow x = \frac{600}{4} = 150$

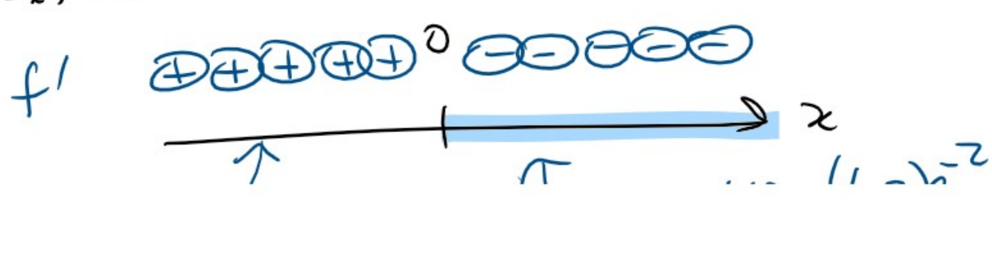
$y = 600 - 2x = 300$

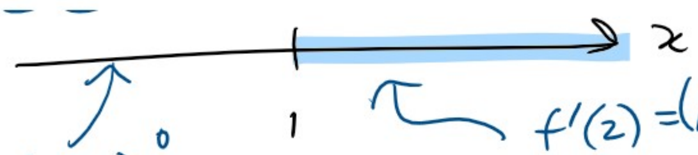
$$\boxed{B}$$

#6: $f(x) = xe^{-x}$ where both: decreasing $\rightarrow f'$ $\ominus$
concave up $\rightarrow f''$ $\oplus$

$f'(x) = 1\cdot e^{-x} + x(-e^{-x}) = (1-x)e^{-x}$

$f'(x) = 0$ when $(1-x) = 0$ $x = 1$





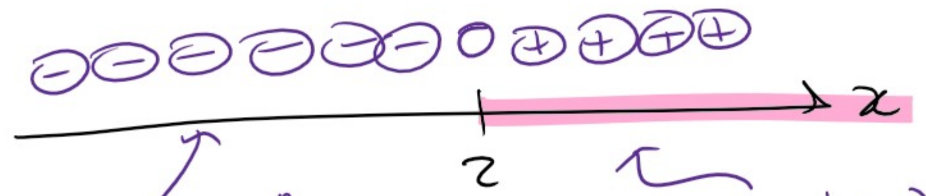
$f'(0) = (1-0)e^0$
$= 1$

$f'(2) = (1-2)e^{-2}$
$= -e^{-2}$
$-$

$\boxed{f \text{ is decreasing on } (1, \infty)}$

$$f''(x) = (-1)e^{-x} + (1-x)(-e^{-x}) = (x-2)e^{-x}$$

$f''(x) = 0$ when $x = 2$



$f''(0) = (0-2)e^0$

$f''(3) = (3-2)e^{-3}$
$+$

$\boxed{f \text{ concave up on } (2, \infty)}$

f is both on $(2, \infty)$ $\boxed{D}$

#7. $\displaystyle \lim_{x \to 0} \frac{1 - \cos(2x)}{5x^3 + 4x^2}$ $\longrightarrow$ $\frac{1 - \cos(0)}{0^2 + 0^2} = \frac{1-1}{0} = \frac{0}{0}$

L'Hopital's

$\overset{L}{=} \displaystyle \lim_{x \to 0} \frac{+\sin(2x)(2)}{15x^2 + 8x}$ $\longrightarrow$ $\frac{2 \cdot \sin(0)}{0} = \frac{0}{0}$ L'Hopital's

$\overset{L}{=} \displaystyle \lim_{x \to 0} \frac{2\cos(2x) \cdot 2}{30x + 8}$ $\longrightarrow$ $\frac{4 \cdot \cos(0)}{8} = \frac{4}{8} = \boxed{\frac{1}{2}}$

$\boxed{B}$

$f(x) = \sqrt{x}$

#8: approx $\sqrt{99.8}$

$f(x) = \sqrt{x}$

$a = 100$

$L(x) = f(a) + f'(a)(x-a)$

$f(a) = \sqrt{100} = 10$

$f'(a) = \left[\frac{1}{2}x^{-1/2}\right]\Big|_{x=100} = \frac{1}{2\sqrt{100}} = \frac{1}{20}$

$L(99.8) = 10 + \frac{1}{20}(99.8 - 100)$

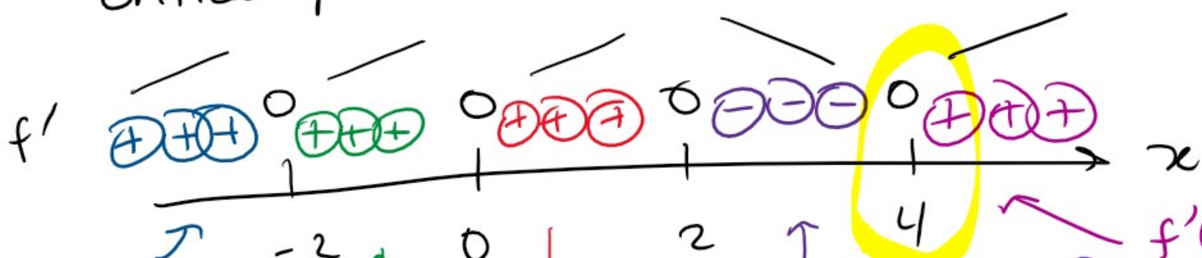
$= 10 + \frac{1}{\cancel{20}}(-0.\cancel{2})$

$= 10 - 0.01 = \boxed{9.99}$ $\boxed{A}$

#9: $f(x)$ whose deriv: $f'(x) = x^2(x-4)(x-2)(x+2)^2$

local minima First Deriv Test

critical points: $f'(x) = 0 \rightarrow x = 0, 4, 2, -2$



$f'(-3) = (-3)^2(-7)(-5)(-1)^2$
$= (+)(-)(-)(+)$
$= (+)$

$f'(-1) = (-1)^2(-5)(-3)(1)^2$
$= (+)(-)(-)(+)$
$= (+)$

$f'(1) = (1)^2(-3)(-1)(3)^2$
$= (+)(-)(-)(+) = (+)$

$f'(3) = 3^2(-1)(1)(5)^2$
$= (+)(-)(+)(+)$
$= (-)$

$f'(5) = 5^2(1)(3)(7)^2$
$= (+)$

one local min @ $x = 4$ $\boxed{A}$

#10: $f(x) = 8x + \frac{6}{x^2}$ $F(x)$ antiderivative

$F(1) = 1$ $F(2) = ?$

$$F(x) = \int \left(8x + \frac{6}{x^2}\right) dx = \frac{8x^2}{2} + 6\left(\frac{x^{-1}}{-1}\right) + C$$

$$F(x) = 4x^2 - \frac{6}{x} + C$$

$$F(1) = 1$$

$$4(1)^2 - \frac{6}{1} + c = 1$$

$$4 - 6 + c = 1$$

$$c = 1 + 2 = 3$$

$\boxed{E}$

$$F(2) = \left(4x^2 - \frac{6}{x} + 3\right)\Big|_{x=2}$$

$$= 4\cdot 2^2 - \frac{6}{2} + 3 = 16 - 3 + 3 = \boxed{16}$$

#11: $f(x) = \frac{x^6}{30} - \frac{x^4}{12}$ # of inflection points

f'' $- \to +$

$+ \to -$

$$f'(x) = \frac{6x^5}{30} - \frac{4x^3}{12} = \frac{x^5}{5} - \frac{x^3}{3}$$

potential infl points

$f''(x) = 0$

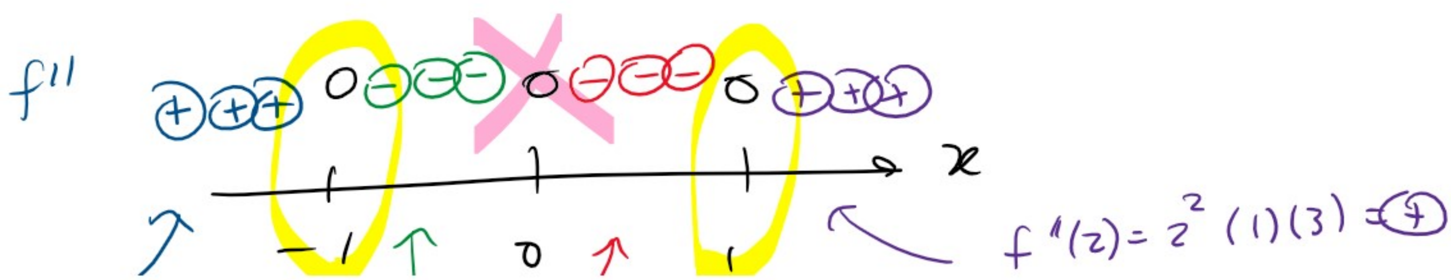
$$f''(x) = \frac{5x^4}{5} - \frac{3x^2}{3} = x^4 - x^2$$

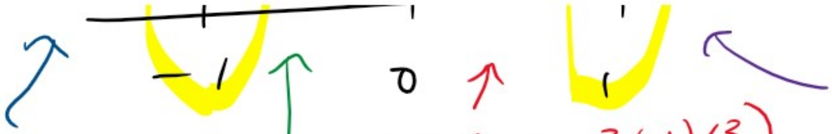
$$f''(x) = 0$$

$$x^4 - x^2 = x^2(x^2 - 1) = 0$$

$$x^2(x-1)(x+1) = 0$$

$$x = 0, 1, -1$$





$f''(2) = 2^2 (1)(3) = +$

$f''(-2) = (-2)^2(-3)(-1)$

$\oplus \ominus \ominus = \oplus$

$f''(\tfrac{1}{2}) = (\tfrac{1}{2})^2(-\tfrac{1}{2})(\tfrac{3}{2})$

$= \oplus \ominus \oplus = \ominus$

$f''(-\tfrac{1}{2}) = (-\tfrac{1}{2})^2(-\tfrac{3}{2})(\tfrac{1}{2})$

$= \oplus \ominus \oplus = \ominus$

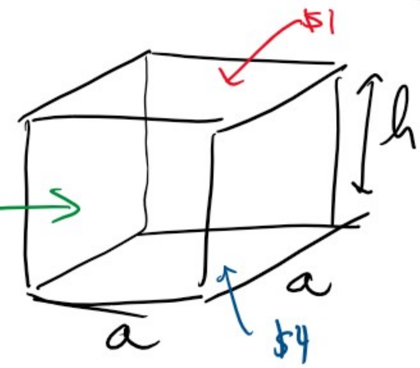
inflection points at $x = -1, 1$

$\boxed{A}$

#12: $V = 10 \text{ ft}^3$

$V = a^2 h = 10$

$h = \dfrac{10}{a^2}$



$C = \$4(a^2) + \$2(4ah) + \$1(a^2)$

min $C = 5a^2 + 8ah$

$C = 5a^2 + 8a \cdot \dfrac{10}{a^2} = 5a^2 + \dfrac{80}{a}$

critical points

$C'(a) = 0$

$10a - \dfrac{80}{a^2} = 0$

$10a = \dfrac{80}{a^2}$

$a^3 = 8$

$\boxed{C}$

$\boxed{a = 2}$

$h = \dfrac{10}{a^2} = \dfrac{10}{4} = \boxed{\dfrac{5}{2}}$