Final Exam from Fall 2018

#9: Half life = 1000 years
initially sample weighs 100g
How much after 100 years.

$y(t) = y_0 e^{kt}$

$y_0 = 100g$
$k < 0$

$T_{1/2} = 1000 = \frac{\ln(\frac{1}{2})}{k}$

$k = \frac{\ln(\frac{1}{2})}{1000} < 0$

$y(t) = 100 e^{\frac{\ln(\frac{1}{2})}{1000} \cdot t}$

@ $t = 100$ years

$y(100) = 100\, e^{\frac{\ln(\frac{1}{2})}{1000} \cdot 100}$

$= 100\, e^{\frac{\ln(\frac{1}{2})}{10}}$

$= 100\, e^{\ln\left(\left(\frac{1}{2}\right)^{\frac{1}{10}}\right)}$ $= 100 \cancel{e}^{\cancel{\ln}(2^{-\frac{1}{10}})}$

$= 100 \cdot 2^{-\frac{1}{10}} = \frac{100}{2^{\frac{1}{10}}} = \frac{100}{\sqrt[10]{2}}$

$\boxed{D}$

#3 $y = \frac{1-2x}{x^3-1}$

horizontal asymptote $y = h$

$h = \lim_{x\to\infty} \frac{1-2x}{x^3-1} \frac{(\frac{1}{x^3})}{(\frac{1}{x^3})}$

$= \lim_{x\to\infty} \frac{\frac{1}{x^3} - \frac{2}{x^2}}{1 - \frac{1}{x^3}} = \frac{0}{1} = 0$

$h = 0$

vertical asymptote $x = k$

$x^3 - 1 = 0$
$x^3 = 1$
$x = 1$

$k = 1$

$h + k = 0 + 1 = 1$

$\boxed{A}$

#11: If $f(x) = \log_{10}(x)$ $f'(e)$

change of base

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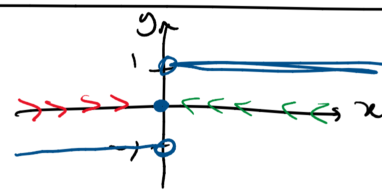
change of base

$$f(x) = \frac{\ln(x)}{\ln(10)} \qquad f'(x) = \frac{1}{\ln(10)} \cdot \frac{1}{x}$$

$$f'(e) = \frac{1}{\ln(10)} \cdot \frac{1}{e}$$

$\boxed{D}$

#4: $f(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$

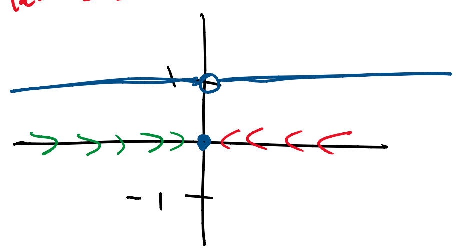


✓ (i) $\lim_{x \to 0^+} f(x) = 1$

✗ (ii) $\lim_{x \to 0} f(x) = \text{DNE}$

$\lim_{x \to 0^-} f(x) = -1$

f(x) is discontinuous @ $x=0$



✓ (iii) $\lim_{x \to 0} |f(x)| = 1$

removable discontinuity

$\boxed{B}$

#6: $\lim_{x \to 2} \sqrt{\frac{x^2-4}{x-2}}$ → $\sqrt{\frac{2^2-4}{2-2}} - \frac{0}{0}$ indeterminate

L'Hopital's Rule works

messy

algebra

$$= \lim_{x \to 2} \sqrt{\frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}} = \lim_{x \to 2} \sqrt{x+2} = \sqrt{4} = 2$$

$\boxed{D}$

#8: $\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{4x}$ → 1^∞ indeterminate

want to use l'Hopital's

$\frac{0}{0}$ or $\frac{\infty}{\infty}$

$\ln\left[\lim_{x \to \infty} \left(1+\frac{3}{x}\right)^{4x}\right]$

$= e^{\lim \ln\left[\left(1+\frac{3}{x}\right)^{4x}\right]}$ $= \lim_{x \to \infty} 4x \ln\left(1+\frac{3}{x}\right)$ → $\infty \cdot 0$

$$= e^{\lim_{x\to\infty} \ln\left[\left(1+\frac{3}{x}\right)^{4x}\right]} = e^{\lim_{x\to\infty} 4x \ln\left(1+\frac{3}{x}\right)} \to \infty \cdot 0$$

indeterminate

$$= e^{\lim_{x\to\infty} \frac{\ln\left(1+\frac{3}{x}\right)}{\left(\frac{1}{4x}\right)}}$$

$\frac{\ln(1)}{0} = \frac{0}{0}$ L'Hopital's Rule

$$\overset{L}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{1+\frac{3}{x}}\cdot\left(\frac{+3}{\cancel{x^2}}\right)}{\left(\frac{+1}{4\cancel{x^2}}\right)}} = e^{\lim_{x\to\infty} \frac{\left(\frac{3}{1+\frac{3}{x}}\right)}{\left(\frac{1}{4}\right)}}$$

$$= e^{\lim_{x\to\infty} \frac{3\cdot 4}{1+\frac{3}{x}}} = e^{12}$$

$\boxed{C}$

#12: $f(x) = (1-x^2)\sin^{-1}(x)$ product Rule

$$f'(x) = (-2x)\sin^{-1}(x) + (1-x^2)\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$= -2x\sin^{-1}(x) + \frac{(1-x^2)^1}{(1-x^2)^{1/2}}$$

$$= -2x\sin^{-1}(x) + (1-x^2)^{1-\frac{1}{2}}$$

$\boxed{B}$

$$= -2x\sin^{-1}(x) + \sqrt{1-x^2}$$

#7: $\lim_{x\to 0} \frac{\csc(3x)}{\cot(x)} = \lim_{x\to 0} \frac{1}{\sin(3x)} \cdot \tan(x)$

L'Hopital's Rule

$$= \lim_{x\to 0} \frac{1}{\sin(3x)} \cdot \frac{\sin(x)}{\cos(x)} \to \frac{0}{0\cdot 1} = \frac{0}{0}$$

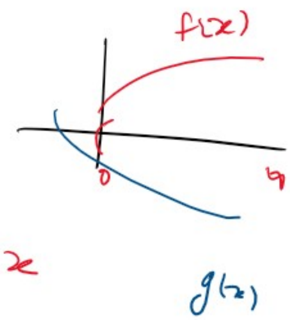
$$\overset{L}{=} \lim_{x\to 0} \frac{\cos(x)}{3\cos(3x)\cdot\cos(x) + \sin(3x)(-\sin(x))} = \frac{1}{3-0}$$

$$= \boxed{\frac{1}{3}}$$

From Chain Rule

$\boxed{A}$

#1: Domain of $f(x)$ is $[0, \infty)$ $y = x+1$



Domain of $g(x) = 1 - f(x+1)$

only affect

$0 \le x+1 < \infty$ solve for x

Domain: ...

only affect Range of g(x)

$0 \le x+1 < \infty$ — solve for x

$-1 \le x < \infty$

$[-1, \infty)$ $\boxed{C}$

$g(x)$

#2: $f(x) = \frac{x+5}{x+1}$ Simplify

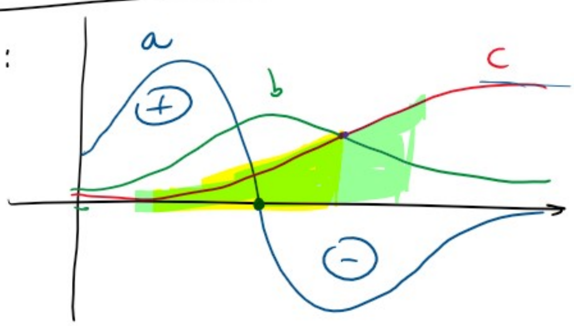
$$\frac{f(x)-f(3)}{x-3} = \frac{\left(\frac{x+5}{x+1}\right) - \left(\frac{3+5}{3+1}\right)}{x-3} = \frac{1}{x-3}\left[\frac{x+5}{x+1} - \cancel{\frac{8}{4}}^{\,2}\right]$$

$$= \frac{1}{x-3}\left[\frac{x+5}{x+1} - 2\left(\frac{x+1}{x+1}\right)\right]$$

$$= \frac{1}{x-3}\left[\frac{x+5-2x-2}{x+1}\right] = \frac{1}{x-3}\left[\frac{-x+3}{x+1}\right]$$

$$= \frac{1}{\cancel{(x-3)}}\left[\frac{-\cancel{(x-3)}}{x+1}\right] = \frac{-1}{x+1} \quad \boxed{B}$$

#5:



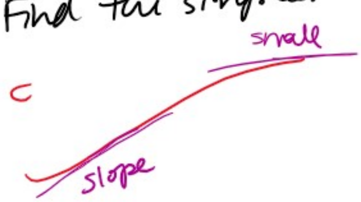
position - $x(t)$ — c

velocity - $v(t) = x'(t)$ — b

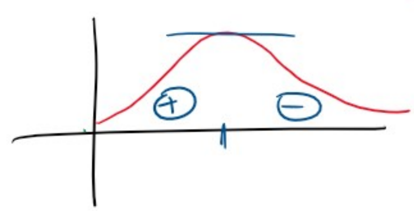
acceleration $a(t) = v'(t)$ — a
$= x''(t)$

$b = \frac{d}{dx}(c)$

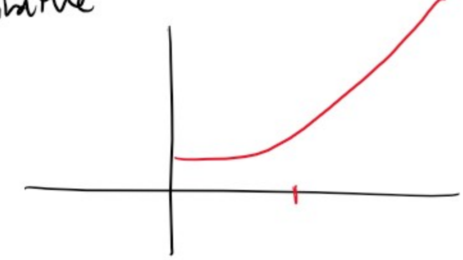
Find the simplest fcn



derivative
pos



anti derivative



$\times$

$\boxed{C}$

$F(x) = \int f(x)$

$F(x)$ is increasing

f' is positive

$F''(x)$ pos ++

concave up

$f''(x)$ pos
concave up

#13: $\frac{d}{dx}\left[f(g(x)) \right] \Big|_{x=2} = f'(g(x)) \cdot g'(x) \Big|_{x=2}$

$g(2) = 3$

$g'(2) = -1$

$f'(3) = 7$

$= f'(g(2)) \cdot g'(2)$

$= f'(3) \cdot (-1)$

$= 7 \cdot (-1) = \boxed{-7}$ B