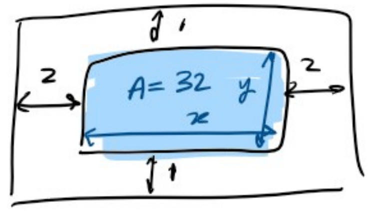
#9: min Total Area T



$A = xy = 32$

$\min \quad T = (x+4)(y+2)$

$y = \frac{32}{x}$

$\min \quad T(x) = (x+4)\left(\frac{32}{x}+2\right) = 32 + 2x + \frac{4 \cdot 32}{x} + 8$

$\min \quad T(x) = 2x + \frac{128}{x} + 40$

critical points: $T'(x) = 0$

$2 + 128\left(\frac{-1}{x^2}\right) = 0 \qquad 2 = \frac{128}{x^2}$

$x^2 = 64 \quad \boxed{x = 8}$

Check its a min: ✓
First Deriv Test

$T'(x)$ − − − − 0 + + + + x

$x=8$

8 local min

$T'(1) = 2 - \frac{128}{1^2} = -$

$T'(9) = 2 - \frac{128}{9^2} = +$

Total Area: $T(8) = 2 \cdot 8 + \frac{128}{8} + 40$

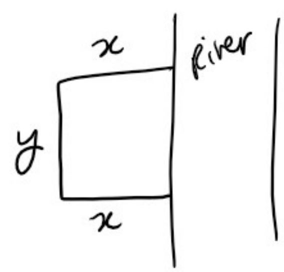
$= 16 + 16 + 40 = \boxed{72}$ $\boxed{A}$

#5: 1000 ft of fence

max $A = xy$

$P = 1000 = 2x + y$

$y = 1000 - 2x$



$\max \quad A(x) = x(1000 - 2x) = 1000x - 2x^2$

critical points: $A'(x) = 0$

$1000 - 4x = 0$

$x = 250$

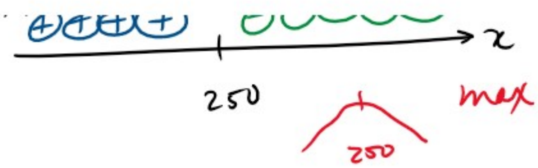
Check if max: First Deriv Test

$A'(x)$ + + + + − − − − − x

250 max

$1000 - 4x = 0$

$x = 250$



$\boxed{x = 250 \qquad y = 500}$

$y = 1000 - 2x = 1000 - 2(250) = 500$

$\boxed{B}$

10: $f(x) = x^{2/3}$ on $[-1, 1]$. Which is FALSE?

A. On $[-1,1]$, f attains an abs min.

critical points: $f'(x) = \frac{2}{3}x^{-1/3} \stackrel{?}{=} 0$

$\frac{2}{3x^{1/3}} \neq 0$

$\frac{2}{3}x^{-1/2} = \text{DNE}$

@ $x=0$

$f'(x)$ undefined

$\boxed{x=0}$

Check for abs min:
evaluate at endpt + critical point:

$f(-1) = (-1)^{2/3} = 1$ ← abs max

$f(0) = 0^{2/3} = 0$ ← abs min @ $x=0$

$f(1) = (1)^{2/3} = 1$ ← abs max

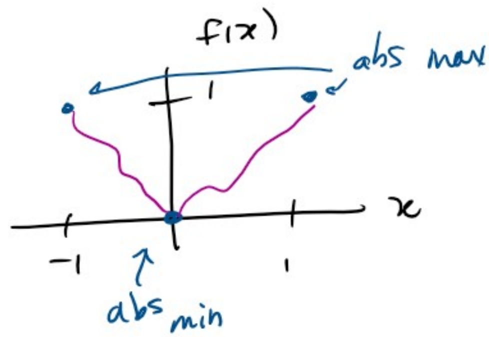
TRUE

B. $x = 0$ is a critical number for f TRUE

$f'(0) = \text{DNE}$ ✓

C. $f(x) \leq 1$ on $[-1, 1]$

TRUE



$f(x) \leq 1$
for all
x in $[-1,1]$

D. There exist a c between 0 and 1 where $f'(c) = 1$

MVT: If $f(x)$ is differentiable, on $[a,b]$

$f'(c) = \frac{f(b) - f(a)}{b - a}$ $\quad b = 1$, $a = 0$ ✓

TRUE

$= \frac{f(1) - f(0)}{1 - 0} = \frac{1 - 0}{1 - 0} = 1$

E. Since $f(-1) = f(1)$, there exist a c between -1 and 1

E. Since $f(-1)=f(1)$, there exist a c between -1 and 1 where $f'(c)=0$

MVT: $f'(c) = \dfrac{f(1)-f(-1)}{1-(-1)}$

$f'(0)$ DNE

MVT Does Not Apply

$\boxed{\text{FALSE}}$

$f'(x) = \dfrac{2}{3x^{1/3}} \overset{?}{\cancel{=}} 0$

#4: Use linear approx to estimate $\sqrt{9.1}$

$f(x) = \sqrt{x}$

at $x=9$ $\quad L(x) = f(a) + f'(a)(x-a)$ ← equation of tangent line

choose a so that $9.1-a$ is small $\quad \boxed{a=9}$

$f(9) = \sqrt{9} = 3$

$f'(9) = \frac{1}{2}x^{-1/2}\Big|_{x=9} = \dfrac{1}{2\sqrt{9}} = \dfrac{1}{6}$

$L(x) = 3 + \frac{1}{6}(x-9)$

$L(9.1) = 3 + \frac{1}{6}(9.1-9) = 3 + \dfrac{0.1}{6}$

$= 3 + \dfrac{1}{60}$ $\quad \boxed{D}$

$(3)\left(\frac{1}{60}\right) = \dfrac{181}{60} = \underbrace{\left(\dfrac{180}{60}\right)}_{3}\left(\dfrac{1}{60}\right) = 3\dfrac{1}{60}$

#2: $\lim\limits_{x\to 0} (1-2x)^{1/x}$ → 1^{∞} indeterminate

L'Hospital's Rule

rearrange $\frac{0}{0}$ or $\frac{\infty}{\infty}$

→ $\ln(\ )$

$\ln\left(\lim\limits_{x\to 0} (1-2x)^{1/x}\right)$ $\qquad \lim\limits_{x\to\infty} \ln\left[(1-2x)^{1/x}\right]$

$$e^{\ln\left(\lim_{x\to 0}(1-2x)^{1/x}\right)} = e^{\lim_{x\to\infty} \ln\left[(1-2x)^{1/x}\right]}$$

$$= e^{\lim_{x\to 0}\frac{\ln(1-2x)}{x}} \rightarrow \frac{\ln(1)}{0} = \frac{0}{0} \quad \text{L'Hopitals}$$

$$\overset{L}{=} e^{\lim_{x\to 0}\frac{\frac{1}{1-2x}\cdot -2}{1}} = e^{\lim_{x\to 0}\frac{-2}{1-2x}} = \boxed{e^{-2}} \quad \boxed{E}$$

#1: $\lim_{x\to\pi} \dfrac{\sin(x-\pi)}{x^2+4\pi x-5\pi^2} \rightarrow \dfrac{\sin(\pi-\pi)}{\pi^2+4\pi^2-5\pi^2} = \dfrac{\sin(0)}{0} = \dfrac{0}{0}$

$$\overset{L}{=} \lim_{x\to\pi}\frac{\cos(x-\pi)}{2x+4\pi} = \frac{\cos(\pi-\pi)}{2\pi+4\pi} = \boxed{\frac{1}{6\pi}} \quad \boxed{A}$$

#12: Find the shape of $y = 3x^4 - 8x^3$

Critical points: $f'(x) = 0$

$12x^3 - 24x^2 = 0$

$12x^2(x-2) = 0$

$x = 0, 2$

First Deriv Test

indeterminate

local min

f': $\ominus\ominus\ominus$ 0 $\ominus\ominus\ominus$ 0 $\oplus\oplus\oplus$ x

0 2

$f'(-1) = 12(-1)^2(-3)$ $\ominus$

$f'(1) = 12(1)^2(-1) = \ominus$

$f'(3) = 12(3)^2(1)$ $\oplus$

Concavity: $f''(x) = 36x^2 - 48x = 0$

$12x(3x-4) = 0$

$x = 0, \frac{4}{3}$

inflection point @ $x=0$

f'': $\oplus\oplus\oplus$ 0 $\ominus\ominus\ominus\ominus$ 0 $\oplus\oplus\oplus$ x

0 $\frac{4}{3}$

$f''(-1) = 12(-1)(-)$

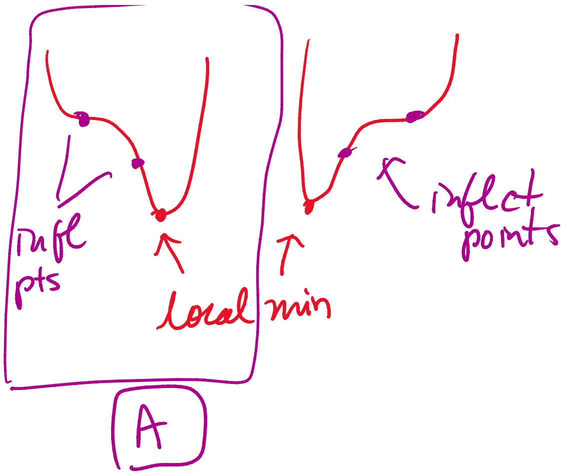
$f''(\frac{1}{2}) = 12(\frac{1}{2})(\frac{3}{2}-4)$ $\oplus\ominus$

~~$f''(1) = 12 \cdot 1(3-4)$~~ $\ominus\ominus = (-)$

1 local min @ $x=2$

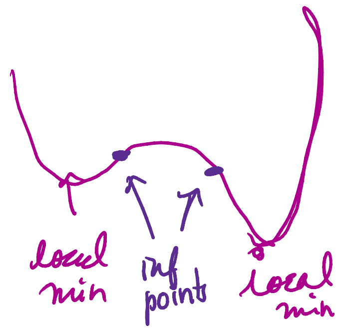
2 inflection points @ $x = 0, \frac{4}{3}$

$f''(2) = 12 \cdot 2\,(6-4)(+)$



$f'(2) = 12 \cdot 2\,(6-4)\,+$

$\cancel{f''(1) = 12 \cdot 1\,(3-4)} = -$



D

X