Series Questions:

F18 #15: The quantity $(\cos 2x)\sum_{n=1}^{\infty}(\tan x)^{2n}$ for $0 \le x < \frac{\pi}{4}$ is equal to ?

← almost Geometric series
need n to start at 0
$k = n-1$ $\quad n = k+1$

$$(\cos 2x)\sum_{n=1}^{\infty}(\tan^2 x)^n$$

$$(\cos 2x)\sum_{k=0}^{\infty}(\tan^2 x)^{k+1} = (\cos 2x)(\tan^2 x)\sum_{k=0}^{\infty}(\tan^2 x)^k$$

Geometric

$$= \frac{(\cos 2x)(\tan^2 x)}{1-\tan^2 x}\left(\frac{\cos^2 x}{\cos^2 x}\right) = \frac{(\cos 2x)\sin^2 x}{\cos^2 x - \sin^2 x}$$

$$= \frac{(\cos 2x)\sin^2 x}{\left(\frac{1+\cos 2x}{2}\right) - \left(\frac{1-\cos 2x}{2}\right)} = \frac{\frac{\cos 2x \sin^2 x}{2}}{\frac{\cos 2x}{2}}$$

$$= \boxed{\sin^2 x}$$