Lesson 12 - MA 166 LEC 286

8.3: Trigonometric Integrals - Part 2

Announcements:

- Exam 1 on Wed Feb 9 @ 6:30-7:30pm
- Review in Class Wednesday
- No Lecture Friday

GOALS:

Evaluate integrals with powers of tan/sec

WARM UP:

Use a trig identity to convert the following integral into one suitable for integration by parts

$$\int [1 - \cos(2x)]^{3/2}\,dx$$

$\boxed{(a)\ 2^{3/2} \int \sin^3(x)\,dx}$

(b) $2^{3/2} \int \cos^3(x)\,dx$

(c) $\int \sin^{3/2}(2x)\,dx$

Half angle formula

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$

$$2\sin^2(x) = 1 - \cos(2x)$$

$$\int [2\sin^2(x)]^{3/2}\,dx$$

$$= 2^{3/2} \int \sin^3(x)\,dx$$

EXAM 1 INFO:

Wed Feb 9 @ 6:30pm-7:30pm in ELLT

- ✩ Be in your seat by 6:15pm
- ✩ Check out seating chart

Bring:
- #2 pencil
- PUID
- Recitatation Section Number
- TAs name

Format:
- Multiple Choice
- 12 Questions - each 8pts

$$12 \times 8 + 4 = 100$$

I. Powers of tan(x) and sec(x)

Review: $\int \sin^m(x)\cos^n(x)\,dx$

m	n	method
odd >0	real #	split off a sin(x) ↳ u-substitution

m		
odd >0	real #	split off a $\sin(x)$ ↳ u-substitution
real #	odd >0	split off a $\cos(x)$ ↳ u-substitution
even ≥ 0	even ≥ 0	Half angle formulas

Today: Want to solve

$$\int \tan^m(x) \sec^n(x)\, dx$$

Tools: u-substition

- $u = \tan(x)$ $\quad du = \sec^2(x)dx$
- $u = \sec(x)$ $\quad du = \sec(x)\tan(x)dx$

Trig Formulas:

$$\frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$\boxed{1 + \tan^2(x) = \sec^2(x)}$$

Ex: Evaluate $\int \tan^3(x) \sec^{\textcircled{4}}(x)dx$

n-even — split off $\sec^2(x)dx = du$

$u = \tan(x)$

$$\int \underbrace{\tan^3(x)}_{u^3} \underbrace{\sec^2(x)}_{\text{rewrite as a fcn of } \tan(x)} \underbrace{\sec^2(x)dx}_{du}$$

$\sec^2(x) = 1 + \tan^2(x)$

$$= \int \underbrace{\tan^3(x)}_{u^3} \underbrace{[1 + \tan^2(x)]}_{1+u^2} \underbrace{\sec^2(x)dx}_{du}$$

$$= \int u^3[1+u^2]du = \int u^3 + u^5 \; du$$

$$= \frac{u^4}{4} + \frac{u^6}{6} + C$$

$$= \boxed{\frac{\tan^4(x)}{4} + \frac{\tan^6(x)}{6} + C}$$

$$= \boxed{\frac{\tan(x)}{4} + \frac{\tan}{6} + C}$$

m	n	method
>0	even ≥ 0	split off $\sec^2(x)$ u-subst. $du = \sec^2(x)dx$
odd >0	>0	?

Ex: $\int \tan^3(x)\sec^3(x)\,dx$

$u = \sec(x)$ $\quad du = \tan(x)\sec(x)dx$

split off $\tan(x)\sec(x)$

$$\int \underbrace{\tan^2(x)}_{} \underbrace{\sec^2(x)}_{u^2} \underbrace{\tan(x)\sec(x)\,dx}_{du}$$

rewrite in terms of sec

$\tan^2(x) = \sec^2(x) - 1$

$$= \int \underbrace{[\sec^2(x)}_{u^2} - 1] \underbrace{\sec^2(x)}_{u^2} \underbrace{\tan(x)\sec(x)\,dx}_{du}$$

$$= \int [u^2-1]u^2\,du = \int u^4 - u^2\,du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= \boxed{\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3} + C}$$

m	n	method
>0	even >0	split off $\sec^2(x)$ ↳ u-subst.
odd >0	>0	split off $\tan(x)\sec(x)$ ↳ u subst. $du = \tan(x)\sec(x)dx$
even	odd	?

Ex: $\int \tan^2(x)\sec(x)\,dx$

$\sec(x)\,dx \quad = \int u\,dv$

Ex: $\int \tan^{2}(x)\sec(x)\,dx$

$$= \int \underbrace{\tan(x)}_{u} \cdot \underbrace{\tan(x)\sec(x)\,dx}_{dv} = \int u\,dv$$

Integration by parts

$u = \tan(x)$ $\qquad dv = \tan(x)\sec(x)\,dx$

$du = \sec^{2}(x)\,dx$ $\qquad v = \int dv = \int \ldots dx$

$= \sec(x)$

IBV

$$= u \cdot v - \int v\,du$$

$$= \tan(x)\cdot\sec(x) - \int \sec(x)\cdot\sec^{2}(x)\,dx$$

$$= \tan(x)\cdot\sec(x) - \int \sec(x)\left[1+\tan^{2}(x)\right]dx$$

$\int \tan^{2}(x)\sec(x)\,dx$

$$= \tan(x)\sec(x) - \int \sec(x)\,dx - \int \tan^{2}(x)\sec(x)\,dx$$

$$2\int \tan^{2}(x)\sec(x)\,dx = \tan(x)\sec(x) - \int \sec(x)\,dx$$

$$= \tan(x)\sec(x) - \ln\left|\sec(x)+\tan(x)\right| + C$$

$$\int \sec(x)\,dx = \int \sec(x)\left(\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}\right)dx$$

$$= \int \frac{\sec^{2}(x)+\tan(x)\sec(x)}{\sec(x)+\tan(x)}\,dx$$

$u = \sec(x)+\tan(x)$

$du = \tan(x)\sec(x) + \sec^{2}(x)$

$$= \int \frac{du}{u} = \ln|u| + C = \ln\left|\sec(x)+\tan(x)\right| + C$$

$$2\int \tan^{2}(x)\sec(x)\,dx = \underbrace{\tan(x)\sec(x) - \ln\left|\sec(x)+\tan(x)\right| + C}$$

$2\int \tan(x)\sec\ldots$

$$\int \tan^2(x)\sec(x)\,dx = \frac{1}{2}\Big[\qquad \Big] + C$$

	m	n	method
★	>0	even >0	split off $\sec^2(x)$ ↳ u-subst.
☆	odd >0	>0	split off $\tan(x)\sec(x)$ ↳ u-subst.
	even >0	odd >0	Integration by Parts

Ex: $\int \tan^3(x)\,dx$

$$= \int \tan(x)\cdot\tan^2(x)\,dx$$

$$= \int \tan(x)\left[\sec^2(x) - 1\right]dx$$

reduction of order

degree dropped

$u = \sec(x)$
$du = \sec(x)\tan(x)$ ⟶

$$= ★\int \underbrace{\tan(x)}_{u=\tan(x)}\underbrace{\sec^2(x)\,dx}_{du} - \int \tan(x)\,dx$$

$$= \int u\,du - \int \sin(x)\left[\cos(x)\right]^{-1}dx \qquad v = \cos(x), \; dv = -\sin(x)dx$$

$$= \int u\,du - \int \frac{-dv}{v}$$

$1 + \tan^2(x) = \sec^2(x)$

$$= \frac{u^2}{2} - \left[-\ln|v|\right] + C$$

$$= \frac{\tan^2(x)}{2} - \left[-\ln|\cos(x)|\right] + C$$

$$= \frac{\tan^2(x)}{2} - \ln\left|\frac{1}{\cos(x)}\right| + C$$

$$= \boxed{\frac{\tan^2(x)}{2} - \ln|\sec(x)| + C}$$

$$= \boxed{\frac{\overset{2}{\tan^{2}}(x)}{2} - \ln|\sec(x)| + C}$$

<u>HW:</u> You will be asked to prove: Reduction of Order Formula

$$\int \tan^{n}(x)\,dx = \frac{\tan^{(n-1)}(x)}{n-1} - \int \tan^{n-2}(x)\,dx$$

where n is an integer $n > 1$