Announcements:
Exam 1 Booklets returned in Recitation on Thursday

8.1 – 8.4
Summary of Trig Integrals + Trig Substitutions

Warm Up: Spring 2013 Exam 2 #1

1. Evaluate the following integral

$$\int_0^{\pi/2} \sin^3 x \cos^2 x \, dx.$$

Trig Integral

A. $\frac{3}{10}$
B. $\frac{1}{10}$
C. $\boxed{\frac{2}{15}}$
D. $\frac{1}{15}$
E. $\frac{\pi}{6}$

$m = 3 \qquad n = 2$

↳ split off a factor of $\sin(x)$

$$\int_0^{\frac{\pi}{2}} \sin^2(x) \cos^2(x) \underbrace{\sin(x)dx}_{du = -\sin(x)dx}$$

$u = \cos(x)$

$$= \int [1 - \cos^2(x)] \cos^2(x) \sin(x) dx$$

$u = \cos(0)$
$u = \cos(\frac{\pi}{2})$

$$= \int_1^0 [1 - u^2] u^2 (-du)$$

$$= \int_1^0 u^4 - u^2 \, du = \left(\frac{u^5}{5} - \frac{u^3}{3}\right]_1^0$$

$$= \left[0 - \frac{1}{5} + \frac{1}{3}\right] = \boxed{\frac{2}{15}}$$

I. Summary:

Trig Integrals:

$$\int \cos^m(x) \sin^n(x) dx$$

m	n	method to solve
	odd	split off $\sin(x)$ $u = \cos(x)$ $du = -\sin(x)dx$
odd		split off a $\cos(x)$ $u = \sin(x)$ $du = \cos(x)dx$
even	even	Half angle formulas $\sin^2(x) = \frac{1 - \cos(2x)}{2}$ $\cos^2(x) = \frac{1 + \cos(2x)}{2}$

$$\int \tan^m(x) \sec^n(x) dx$$

$$\int \tan^m(x)\sec^n(x)\,dx$$

m	n	method to solve
	even	Split off $\sec^2(x)$ $u = \tan(x)$ $du = \sec^2(x)\,dx$ $\sec^2(x) = \tan^2(x)+1$
odd	>0	Split off $\tan(x)\sec(x)$ $u = \sec(x)$ $du = \tan(x)\sec(x)\,dx$ $\tan^2(x) = \sec^2(x)-1$
even	odd	Integration by parts Look for an easy dv $\int u\,dv = u\cdot v - \int v\,du$

Trig Substitutions:

integral	substitution	simplified
$\sqrt{a^2-x^2}$	$x = a\sin\theta$ $dx = a\cos\theta\,d\theta$	$\sqrt{a^2-x^2} \to a\cos\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$ $dx = a\sec^2\theta\,d\theta$	$\sqrt{a^2+x^2} \to a\sec\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$ $dx = a\tan\theta\sec\theta$	$\sqrt{x^2-a^2} \to a\tan\theta$

☆ **NOTE:** May need to complete the square first:

Ex: $\int\sqrt{x^2+bx+c}$

want to use $x = a\tan\theta$

$\sqrt{x^2+a^2}$

Complete the square

$\cancel{x^2}+\cancel{bx}+c = \cancel{x^2}+\cancel{bx}+\frac{b^2}{4}+d \;\to\; c = \frac{b^2}{4}+d$

$= \left(x+\frac{b}{2}\right)^2 + d$

$d = c-\frac{b^2}{4}$

Complete the sq.

$$x^2 + bx + c = \left(x+\frac{b}{2}\right)^2 + d \qquad d = c - \frac{b}{4}$$

$$\rightarrow d = c - \frac{b^2}{4} = a^2$$

$$\int \sqrt{\left(x+\frac{b}{2}\right)^2 + \left(c - \frac{b^2}{4}\right)}$$

u-substitution $\sqrt{u^2+a^2}$

$$u = x + \frac{b}{2} \qquad du = dx$$

$$= \sqrt{u^2 + \left(c - \frac{b^2}{4}\right)}$$

$$a = \sqrt{c - \frac{b^2}{4}}$$

$$u = a\tan\theta$$

☆ Spring 2013 Exam 2 #2

2. Evaluate the following integral

$$\int_{\pi/4}^{\pi/3} \frac{\sec^4 x}{\tan^2 x}\,dx.$$

$$\tan^{-2}(x)\sec^2(x)\underbrace{\sec^2(x)\,dx}_{du}$$

A. $\frac{2}{\sqrt{3}}$

B. $\frac{\sqrt{3}-1}{2}$

C. $\frac{2}{\sqrt{3}}+1$

D. $2\sqrt{3}-1$

E. $2\sqrt{3}$

u-substitution

$$u = \tan(x) \qquad du = \sec^2(x)\,dx$$

$$\int_1^{\sqrt{3}} u^{-2}\left(u^2+1\right)du$$

@ $x = \frac{\pi}{4}$ $\qquad \frac{\pi}{4} = \tan(x)$

$$x = \tan^{-1}\left(\frac{\pi}{4}\right) = 1$$

@ $x = \frac{\pi}{3}$ $\qquad \frac{\pi}{3} = \tan(x)$

$$= \int_1^{\sqrt{3}} 1 + u^{-2}\,du = \left[u + \frac{u^{-1}}{-1}\right]_1^{\sqrt{3}}$$

$$= \boxed{\frac{2}{\sqrt{3}}} \quad \boxed{A}$$

Spring 13 Exam 2 #3

3. Compute

$$\int \frac{dx}{\sqrt{1+4x^2}}.$$

A. $\ln|x+\sqrt{1+4x^2}|+C$

B. $\frac{1}{2}\ln|2x+\sqrt{1+4x^2}|+C$

C. $\frac{1}{2}\tan^{-1}(2x)+\sqrt{1+4x^2}+C$

D. $\tan^{-1}(2x)+\sqrt{1+4x^2}+C$

E. $\frac{1}{2}\ln|1+4x^2|+C$

$$\int \frac{dx}{2\sqrt{\frac{1+4x^2}{4}}} = \frac{1}{2}\int \frac{dx}{\sqrt{(\frac{1}{2})^2+x^2}}$$

$x = \frac{1}{2}\tan\theta$

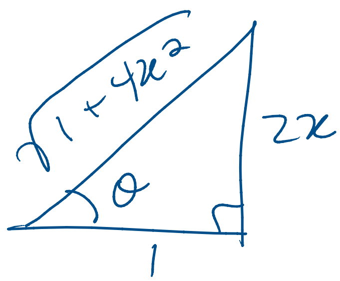
$dx = \frac{1}{2}\sec^2\theta\, d\theta$

$$= \frac{1}{2}\int \frac{\cancel{\frac{1}{2}}\sec^{\cancel{2}}\theta\, d\theta}{\cancel{\frac{1}{2}}\cancel{\sec\theta}}$$

$$= \frac{1}{2}\int \sec\theta\, d\theta$$

$\tan\theta = 2x$

$$= \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$



$\sec\theta = \frac{\sqrt{1+4x^2}}{1}$

$$= \frac{1}{2}\ln|2x+\sqrt{1+4x^2}| + C$$

$\boxed{B}$