Lesson 15 - MA 166 LEC 286

8.1 – 8.4

Summary of Trig Integrals + Trig Substitutions

Announcements:

Exam 1 Booklets returned in Recitation on Thursday

Masks still required in classes

Warm Up: Spring 2013 Exam 2 #1

1. Evaluate the following integral

$$\int_0^{\pi/2} \sin^3 x \cos^2 x \, dx.$$

A. $\frac{3}{10}$

B. $\frac{1}{10}$

C. $\boxed{\frac{2}{15}}$

D. $\frac{1}{15}$

E. $\frac{\pi}{6}$

$u = \cos(x) \qquad du = -\sin(x)\,dx$

$\int_1^0 [1-u^2]\, u^2 \, (-du)$

$= \int_1^0 (u^4 - u^2)\,du = \left[\frac{u^5}{5} - \frac{u^3}{3}\right]_1^0$

$= \left[0 - \frac{1}{5} + \frac{1}{3}\right] = \frac{5-3}{15} = \boxed{\frac{2}{15}} \; \boxed{C}$

I. Summary:

Trig Integrals $\int \cos^m(x) \sin^n(x)\,dx$

m	n	method to solve
	odd	split off a $\sin(x)$ $u = \cos(x)$ $du = -\sin(x)\,dx$ $\sin^2(x) = 1 - \cos^2(x)$
odd		split off a $\cos(x)$ $u = \sin(x)$ $du = \cos(x)\,dx$ $\cos^2(x) = 1 - \sin^2(x)$

even	even	Half angle formulas $\sin^2(x) = \frac{1-\cos(2x)}{2}$ $\cos^2(x) = \frac{1+\cos(2x)}{2}$

$$\int \tan^m(x)\sec^n(x)\,dx$$

m	n	method to solve
	even	split off $\sec^2(x)$ $u = \tan(x)$ $du = \sec^2(x)\,dx$ $\sec^2(x) = 1 + \tan^2(x)$
odd	> 0	split off $\tan(x)\sec(x)$ $u = \sec(x)$ $du = \tan(x)\sec(x)\,dx$ $\tan^2(x) = \sec^2(x) - 1$
even	odd	Integration by Parts $\int u\,dv = u \cdot v - \int v\,du$ __want__: choose dv so that its easy to integrate

Trig Substitutions:

integral	substitution	simplified
$\sqrt{a^2-x^2}$	$x = a\sin\theta$ $dx = a\cos\theta\,d\theta$	$\sqrt{a^2-x^2} \to a\cos\theta$

$\sqrt{a^2-x^2}$	$x = a\sin\theta$ $dx = a\cos\theta\, d\theta$	$\sqrt{a^2-x^2} \to a\cos\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$ $dx = a\sec^2\theta\, d\theta$	$\sqrt{a^2+x^2} \to a\sec\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$ $dx = a\tan\theta\sec\theta\, d\theta$	$\sqrt{x^2-a^2} \to a\tan\theta$

<u>NOTE:</u> May need to complete the square first

<u>Ex</u>: $\int \sqrt{x^2+4x+13}\, dx$

Want: $\sqrt{u^2+a^2}$

Complete the square

$$\cancel{x^2} + \underbrace{\cancel{4x}}_{\frac{4}{2}} + 13 = (x+2)^2 + d$$

$$= \cancel{x^2} + \cancel{4x} + 4 + d$$

$$d = 13 - 4 = 9$$

$$= \int \sqrt{(x+2)^2+9}\, dx$$

u-substitution
$u = x+2$
$du = dx$

$$= \int \sqrt{u^2+3^2}$$

Trig substitution
$u = 3\tan\theta$

Spring 2013 Exam 2 #2

2. Evaluate the following integral

even

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$$\int_{\pi/4}^{\pi/3} \frac{\sec^4 x}{\tan^2 x} dx.$$

even

$\boxed{\text{A. } \frac{2}{\sqrt{3}}}$

B. $\frac{\sqrt{3}-1}{2}$

C. $\frac{2}{\sqrt{3}}+1$

D. $2\sqrt{3}-1$

E. $2\sqrt{3}$

$u = \tan x \qquad du = \sec^2(x)dx$

$\int \tan^{-2}(x)\left[1+\tan^2(x)\right]\sec^2(x)\,dx$

$= \int_1^{\sqrt{3}} u^{-2}\left[1+u^2\right] du$

$u = \tan\left(\frac{\pi}{3}\right) \rightarrow \sqrt{3}$

$u = \tan\left(\frac{\pi}{4}\right) \rightarrow 1$

$= \int_1^{\sqrt{3}} \left[1+u^{-2}\right] du = \left[u + \frac{u^{-1}}{-1}\right]_1^{\sqrt{3}}$

$= \sqrt{3} - \frac{1}{\sqrt{3}} \cancel{-1} \cancel{+1}$

$= \frac{\sqrt{3}\cdot\sqrt{3}}{\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{3-1}{\sqrt{3}} = \boxed{\frac{2}{\sqrt{3}}}$

Spring 2013 Exam 2 #3

3. Compute

$$\int \frac{dx}{2\sqrt{\frac{1+4x^2}{4}}}.$$

could do
$u = 2x$
$du = 2dx$
first

A. $\ln|x+\sqrt{1+4x^2}|+C$

$\boxed{\text{B. } \frac{1}{2}\ln|2x+\sqrt{1+4x^2}|+C}$

C. $\frac{1}{2}\tan^{-1}(2x)+\sqrt{1+4x^2}+C$

D. $\tan^{-1}(2x)+\sqrt{1+4x^2}+C$

E. $\frac{1}{2}\ln|1+4x^2|+C$

$= \frac{1}{2}\int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2+x^2}}$

$x = \frac{1}{2}\tan\theta$

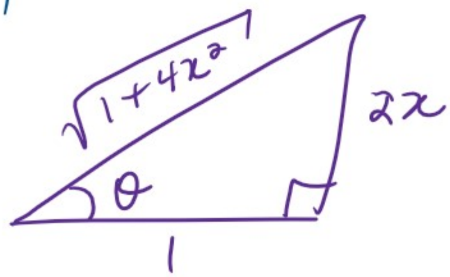
$dx = \frac{1}{2}\sec^2\theta\, d\theta$

$$= \frac{1}{2}\int \frac{\cancel{\frac{1}{2}}\sec^{\cancel{2}}\theta \, d\theta}{\cancel{\frac{1}{2}}\cancel{\sec\theta}} = \frac{1}{2}\int \sec\theta \, d\theta$$

$\tan\theta = 2x$

$$= \frac{1}{2}\ln\left|\tan\theta + \sec\theta\right| + C$$

Ref. Δ



$\tan\theta = 2x$

$\sec\theta = \dfrac{\sqrt{1+4x^2}}{1}$

$$= \boxed{\frac{1}{2}\ln\left|2x + \sqrt{1+4x^2}\right| + C}$$

$\boxed{B}$