Lesson 1 - MA 166 LEC 200

13.1 Vectors in the Plane (2D)

13.2 Vectors in 3D

Announcements:

- No HW / Quiz for Week 1

Warm up:

Before class starts, introduce yourself to your neighbors:

- name
- major
- year
- favorite differentiation rule

Ex:

- Dr. Hood
- professor in the Math Dept
- At Purdue for 2 years
- Product Rule

$[fg]' = f'g + fg'$

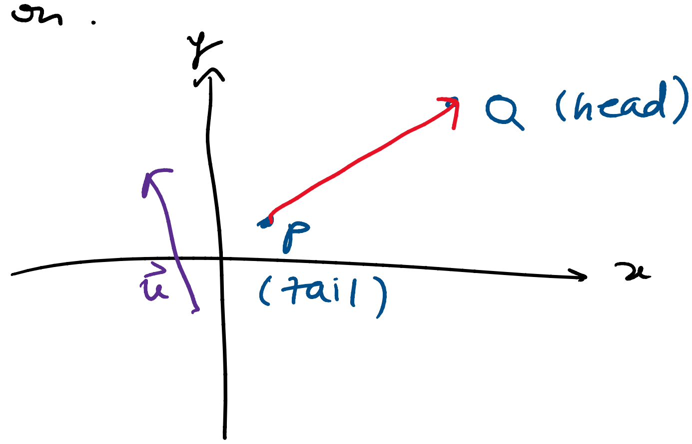
GOALS:

- syllabus
- vector operations
- solve applications using vectors

II. Vectors in 2D

Def: vectors are quantities that have both a length (magnitude) and direction.

$\overrightarrow{PQ}$ vector that starts at P and ends at Q



NOTATION:

We also denote vectors by single letters w/ arrow

$\vec{u}$ is a vector

$\vec{u}$ is a vector

Def: Two vectors are equal if they have the same magnitude + same direction

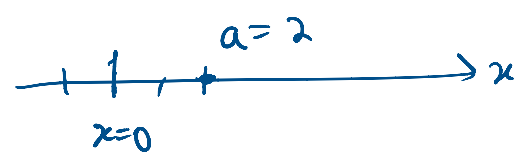
$\vec{PQ} = \vec{P'Q'}$ $\qquad$ $\vec{PQ}$ is parallel to $\vec{P'Q'}$

Def: The zero vector $\vec{0}$ has length 0 and no direction.

Def: A scalar is a quantity with a magnitude and no direction

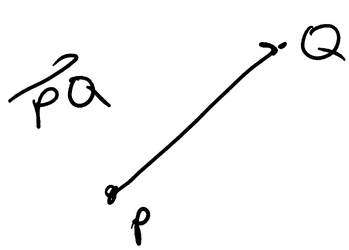
Ex: $a = 2$

π

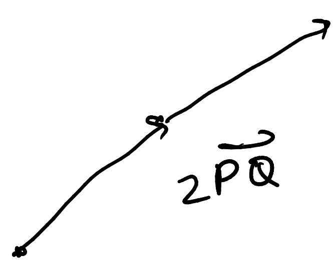


III. Operations on Vectors:

- scalar multiplication $\qquad$ scalar a times vector $\vec{v}$

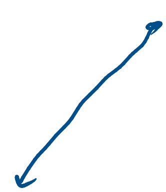


$2\vec{PQ}$



2x length
same direction

$-\vec{PQ}$

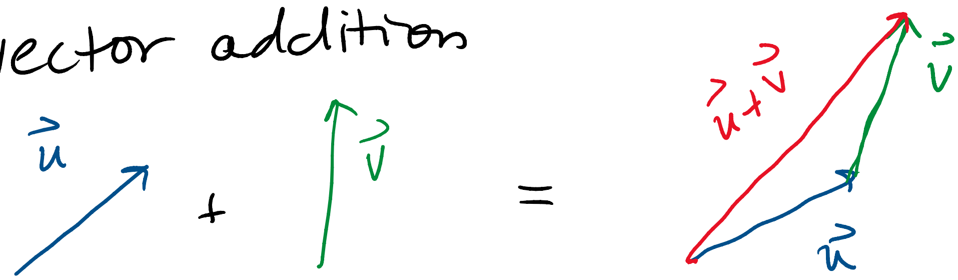


$-\vec{PQ} = (-1)\vec{PQ}$ $\qquad$ same length opposite direction

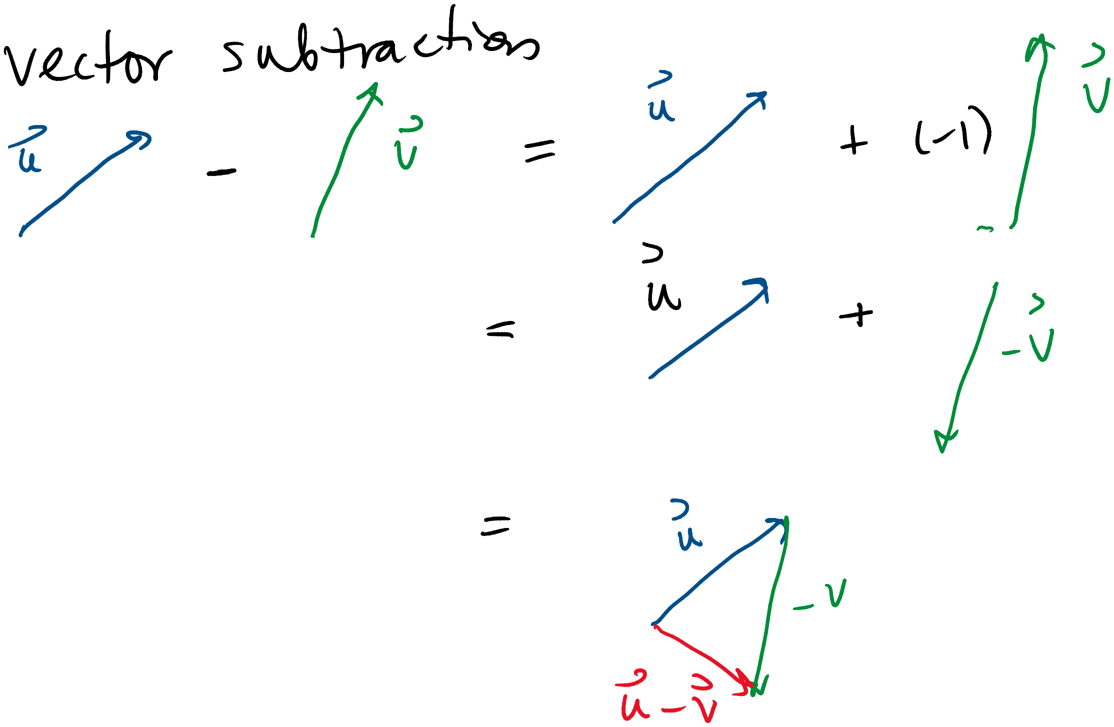
NOTE: two vectors are parallel if they

NOTE: two vectors are parallel if they are scalar multiples of each other

- vector addition



- vector subtraction



<u>Properties:</u> (full list on p811 of textbook)

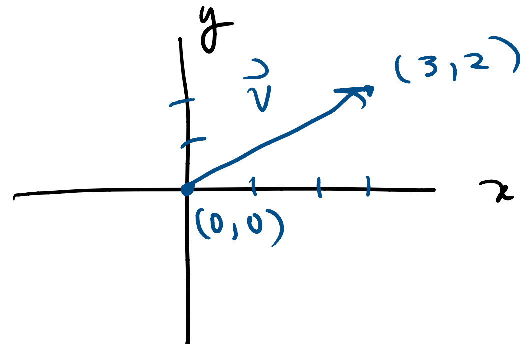
1. $\vec{u}+\vec{v} = \vec{v}+\vec{u}$
2. $\vec{v}+\vec{0} = \vec{v}$ $\quad(\vec{v}-\vec{v}=\vec{0})$
3. $c(\vec{u}+\vec{v}) = c\vec{u}+c\vec{v}$
4. $0\vec{v} = \vec{0}$
5. $a(c\vec{v}) = (ac)\vec{v}$

III. Vector Components:

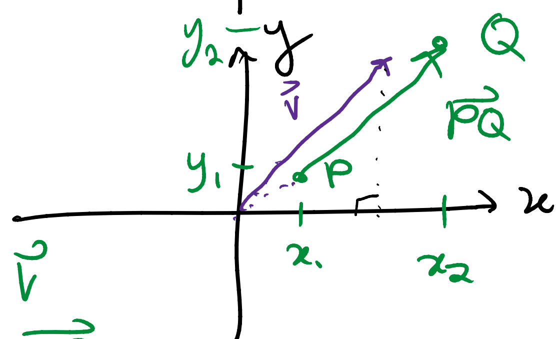
IV. Vector components:

Def: A vector $\vec{v}$ with its at $(0,0)$ and head at (v_1, v_2) is called a *position vector* and is written $\langle v_1, v_2 \rangle$

$\vec{v} = \langle 3, 2 \rangle$ is a position vector



Ex: let $P = (x_1, y_1)$
$Q = (x_2, y_2)$



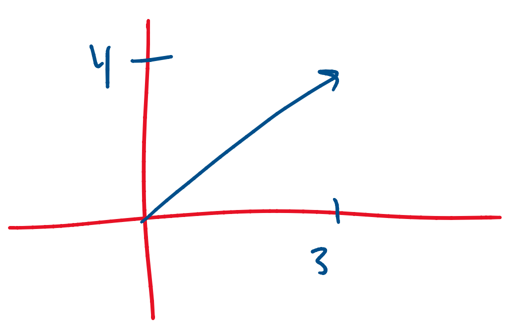
the position vector $\vec{v}$ that is equal to $\vec{PQ}$

$\vec{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$

Def: the magnitude of $\vec{v}$

$$|\vec{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

NOTE: magnitude is a scalar



$\vec{v} = \langle 3, 4 \rangle$

$|\vec{v}| = \sqrt{3^2 + 4^2}$

$= \sqrt{25} = 5$

✯ Vector operations in $\mathbb{R}^2$:

Let $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$

c is scalar

$$\vec{u} + \vec{v} = \langle u_1 + v_1 , u_2 + v_2 \rangle$$

$$\vec{u} - \vec{v} = \langle u_1 - v_1 , u_2 - v_2 \rangle$$

$$c\vec{u} = \langle cu_1 , cu_2 \rangle$$

Def: A unit vector is a vector with length 1

Given a vector $\vec{v}$, we can find the unit vector $\vec{u}$ that is parallel to $\vec{v}$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|}$$

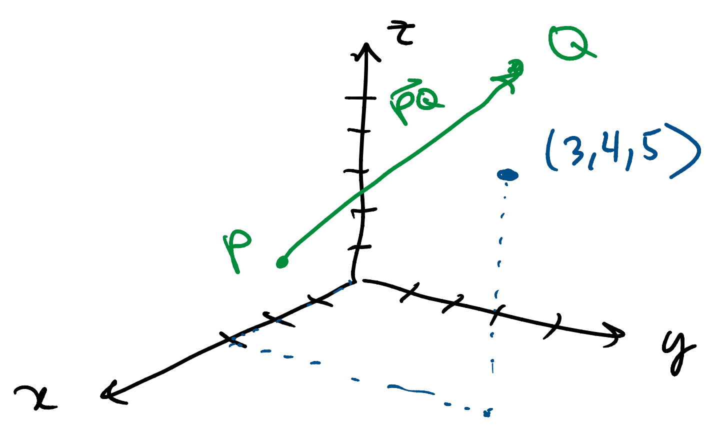
$\frac{1}{|\vec{v}|} = c$ is a scalar

V. Vectors in 3D:

xyz - coordinates

plot point

$\langle 3, 4, 5 \rangle$



Vectors in 3D:

$P = (x_1, y_1, z_1)$ $\qquad$ $Q = (x_2, y_2, z_2)$

$$\vec{PQ} = \langle x_2 - x_1 , y_2 - y_1 , z_2 - z_1 \rangle$$

$\overrightarrow{PQ} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

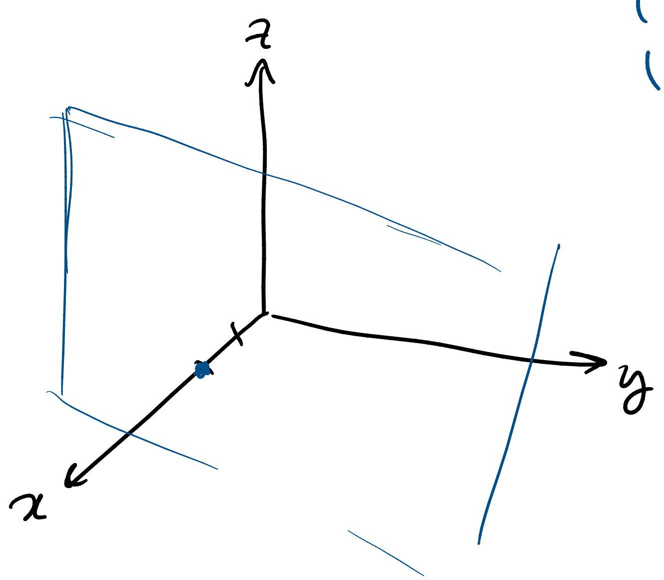
$|\overrightarrow{PQ}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

Equations of simple planes

plane $x = 2$ $\quad \{(x,y,z) : x = 2\}$

the set of all points (x,y,z) where $x=2$

includes $(2,0,0)$
$(2,1,2)$
$(2,-5,-7)$



parallel to the yz-plane

☆ Summary:

- a vector has magnitude + direction
- two vectors are parallel if they are scalar multiples
- $|\vec{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2}$
- unit vector $\vec{u} = \dfrac{\vec{v}}{|\vec{v}|}$