13.1 Vectors in the Plane (2D)

13.2 Vectors in 3D

Announcements:

- No HW / Quiz for Week 1

Warm up:

Before class starts, introduce yourself to your neighbors:

- name
- major
- year
- favorite differentiation rule

Ex:

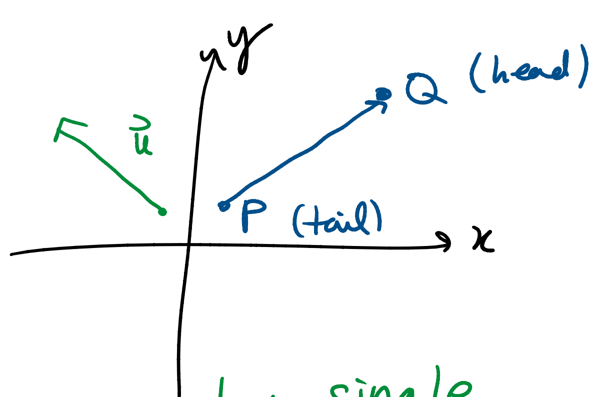
- Dr. Hood
- professor in the Math Dept
- At Purdue for 2 years
- Product Rule
  $[fg]' = f'g + fg'$

GOALS:

- syllabus
- vector operations
- solve applications using vectors

I. Vectors in 2D:

Def: vectors are quantities that have both a "magnitude" and a "direction"
"length"



$\overrightarrow{PQ}$ vector that starts at P and ends at Q

NOTE: We also denote vectors by single lower case letters w/ an arrow $\vec{u}$

equal if they have

lower case letters w/ an ... $\vec{u}$

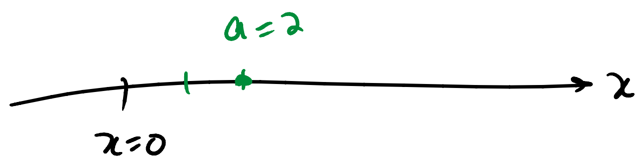
Def: Two vectors are <u>equal</u> if they have the same magnitude + same direction

Def: The zero vector $\vec{0}$ has length 0 and no direction

Def: A <u>scalar</u> is a quantity with a magnitude and no direction

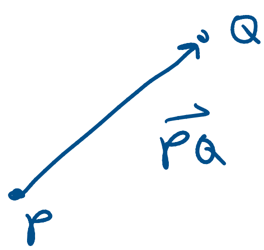
Ex: $a = 2$

π

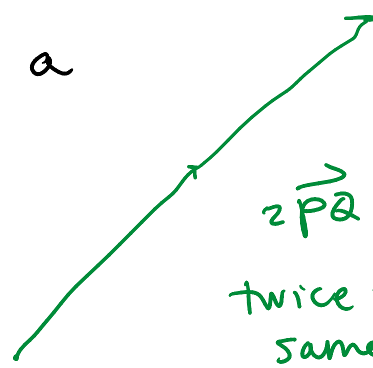


III. operations on vectors

- scalar multiplication $\rightarrow$ $a\vec{v}$
  vector $\vec{v}$ scalar a



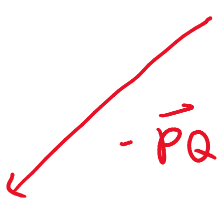
$2\vec{PQ}$



$2\vec{PQ}$

twice the length
same direction

$-\vec{PQ} = (-1)\vec{PQ}$

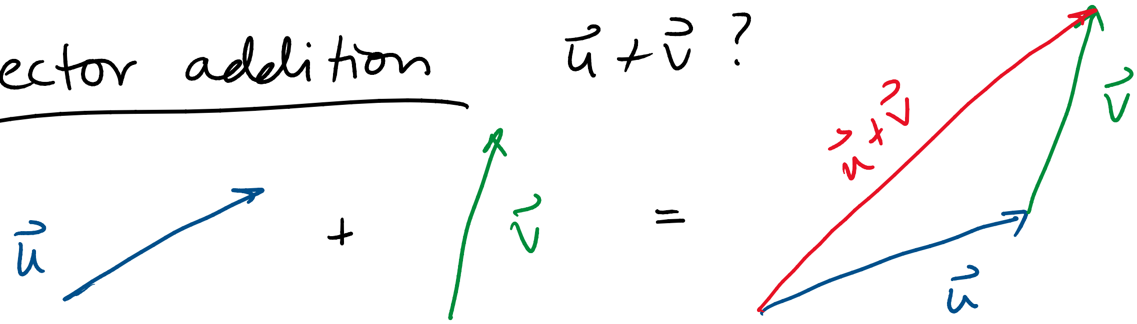


same length
opposite direction

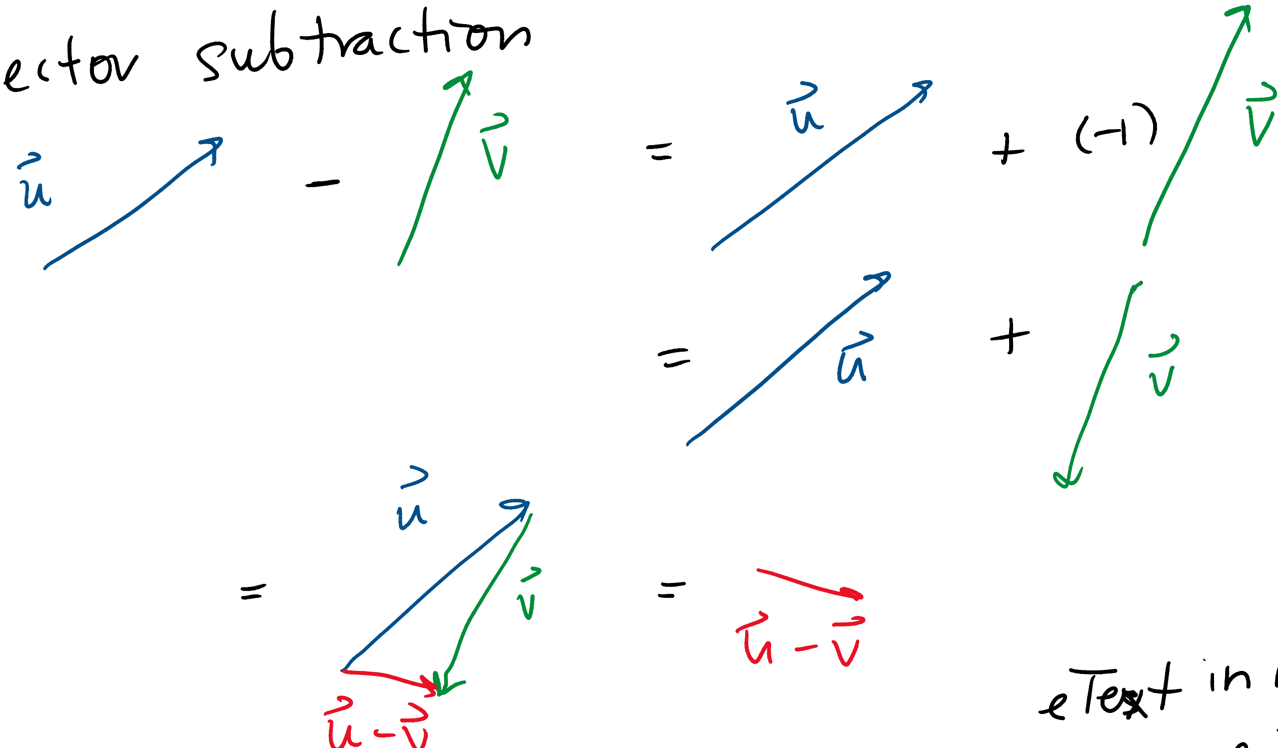
NOTE: two vectors are <u>parallel</u> if they are

NOTE: two vectors are <u>parallel</u> if they are scalar multiples of each other

- <u>vector addition</u> $\vec{u}+\vec{v}$?



- vector subtraction



eText in MIM

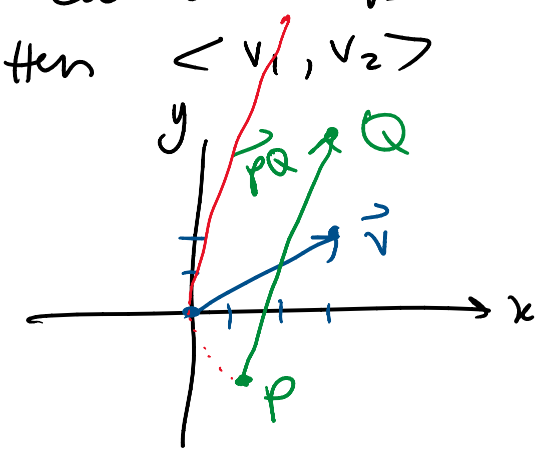
<u>Properties:</u> (full list p. 811 of textbook)

1. $\vec{u}+\vec{v}=\vec{v}+\vec{u}$
2. $\vec{v}+\vec{0}=\vec{v}$ $\quad(\vec{v}-\vec{v}=\vec{0})$
3. $c(\vec{u}+\vec{v})=c\vec{u}+c\vec{v}$
4. $0\vec{v}=\vec{0}$
5. $a(c\vec{v})=(ac)\vec{v}$

3.

IV. Vector Components:

<u>Def</u>: A vector $\vec{v}$ with its tail at $(0,0)$ and head at (v_1, v_2) is called a <u>position vector</u> and is written $\langle v_1, v_2 \rangle$



$\vec{v} = \langle 3, 2 \rangle$ is a position vector

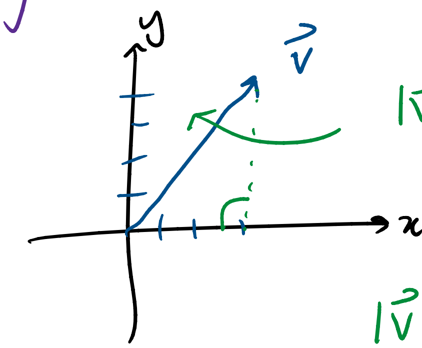
<u>Ex</u>: let $P = (x_1, y_1)$
$Q = (x_2, y_2)$

the position vector $\vec{PQ} = \langle x_2 - x_1, y_2 - y_1 \rangle$

<u>Def</u>: the magnitude of $\vec{PQ}$ is

$$|\vec{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

<u>NOTE</u>: the magnitude is a scalar



$|\vec{v}|$ is the hypotenuse of the triangle

$|\vec{v}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

☆ Vector Operations in 2D:

let $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$
c be a scalar

let $\vec{u} = \langle u_1, u_2 \rangle$ ...

c be a scalar

$$\vec{u} + \vec{v} = \langle u_1 + v_1,\ u_2 + v_2 \rangle$$

$$\vec{u} - \vec{v} = \langle u_1 - v_1,\ u_2 - v_2 \rangle$$

$$c\vec{v} = \langle c u_1,\ c u_2 \rangle$$

Def: A unit vector is a vector with length 1.

Given a vector $\vec{v}$, we can find the unit vector $\vec{u}$ that is parallel to $\vec{v}$

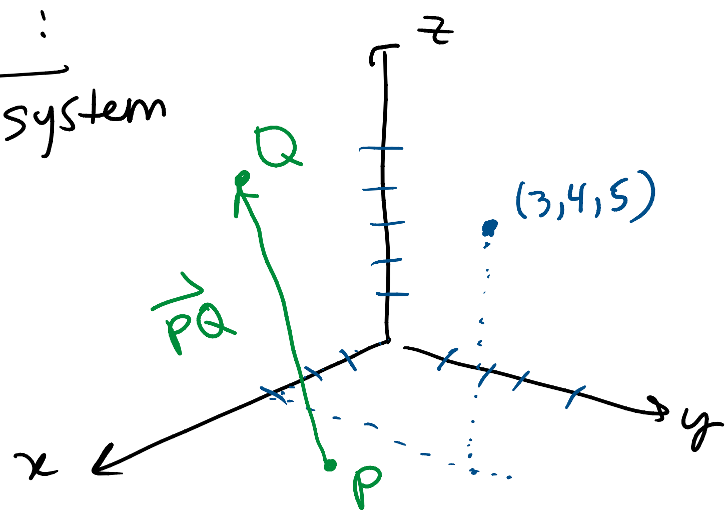
$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} \qquad \frac{1}{|\vec{v}|} = c \text{ is a scalar}$$

V. Vectors in 3D :

xyz - coordinate system

plot point $(3,4,5)$

$x=3,\ y=4,\ z=5$



vectors in 3D

$P = (x_1, y_1, z_1)$ $\qquad$ $Q = (x_2, y_2, z_2)$

$$\vec{PQ} = \langle x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1 \rangle$$

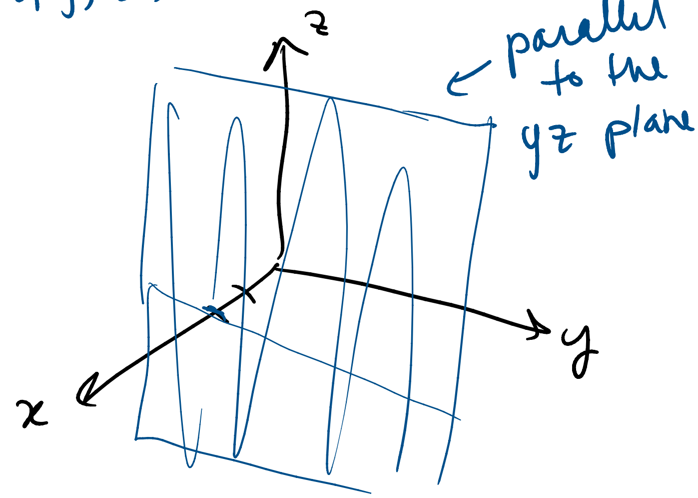
$$|\vec{PQ}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Equations of simple planes in 3D:

plane $x=2$ $\{(x,y,z) : x=2\}$

the set of all points (x,y,z) where $x=2$

includes $(2,0,0)$
$(2,1,0)$
$(2,-7,21)$



POLL:

$$\vec{u} + 2\vec{v} = \langle 1,2,0\rangle + 2\langle 0,1,1\rangle$$
$$= \langle 1,2,0\rangle + \langle 0,2,2\rangle$$
$$= \langle 1,4,2\rangle$$

☆ Summary:

- a vector has a magnitude + direction
- two vectors are parallel if they are scalar multiples
- magnitude $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
- unit vector $\vec{u} = \dfrac{\vec{v}}{|\vec{v}|}$