Lesson 21 - MA 166 LEC 200

10.4: Divergence Test

Announcements:

Exam 2 on Wed Mar 9 @ 6:30pm
Assigned Seats

Review in Class on Wed

Warm Up: Fall 2015 Exam 2 Question #11

(This type of problem will be on exam 3)

11. $\sum_{k=1}^{\infty} \frac{2^k}{3^{k+1}} =$

A. 2/3

B. 1/2

C. 2/9

D. 3/4

E. 4/3

$$\sum_{k=1}^{\infty} \frac{2^k}{3^{k+1}} = \boxed{\frac{2^1}{3^2}} + \boxed{\frac{2^2}{3^3}} + \frac{2^3}{3^4}$$

$$= \boxed{a} + \boxed{ar} + ar^2 + \dots$$

$$a = \frac{2}{9} \qquad ar = \frac{2^2}{3^3} = \underbrace{\left(\frac{2}{9}\right)}_{a}\underbrace{\left(\frac{2}{3}\right)}_{r}$$

$$= \sum_{k=0}^{\infty} \frac{2}{9}\left(\frac{2}{3}\right)^k \qquad |r| = \left|\frac{2}{3}\right| < 1$$

$$= \frac{a}{1-r} = \frac{\frac{2}{9}}{1-\frac{2}{3}} = \frac{\frac{2}{9}}{1/3} = \frac{2}{9}\cdot\frac{3}{1}$$

$$= \boxed{\frac{2}{3}} \ \boxed{A}$$

Wed - Exam 2 Review

→ Hot Seat Poll

Exam 2: lessons 10 – 18 (Integration Techniques)

Book: 8.1–8.5, 8.9

Q: Arc Length? → NO

(Similar format to HW Qs, maybe find area of curves, volume → convert into trig int / trig subst)

Not covered: sequences + series on exam 3

I. Divergence Test:

Q: When does a sequence diverge?

Thm: (Divergence Test)

If $\sum_{k=1}^{\infty} a_k$ converges, then $\lim_{k\to\infty} a_k = 0$

OR

If $\lim_{k\to\infty} a_k \neq 0$, then $\sum_{k=1}^{\infty} a_k$ diverges

NOTE: this only tells us when a series diverges

Ex: (1) $\sum_{k=1}^{\infty} \frac{k}{k+1}$, then $a_k = \frac{k}{k+1}$

$$\lim_{k\to\infty} \frac{k}{k+1} = \lim_{k\to\infty} \frac{1}{1+\frac{1}{k}} = 1 \cancel{\neq} 0$$

the series diverges (by the Divergence Test)

Ex: (2) $\sum_{k=1}^{\infty} \frac{1+3^k}{2^k}$, then $a_k = \frac{1+3^k}{2^k}$

$$\lim_{k\to\infty} \frac{1+3^k}{2^k} = \lim_{k\to\infty} \underbrace{\frac{1}{2^k}}_{\to 0} + \underbrace{\left(\frac{3}{2}\right)^k}_{\to +\infty} = +\infty$$

By the Div. Test, the series diverges

Ex (3): $\sum_{k=1}^{\infty} \frac{1}{k}$ $\qquad a_k = \frac{1}{k}$

$$\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{1}{k} = 0$$

Inconclusive: ← Div Test can't prove that a series converges (in fact $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges)

Ex (4): $\sum_{k=1}^{\infty} \frac{1}{k^2}$ $\qquad a_k = \frac{1}{k^2}$

$\lim_{k\to\infty} \frac{1}{k^2} = 0$ $\qquad$ Divergence Test is inconclusive

(In fact, $\sum_{k=1}^{\infty} \frac{1}{k^2}$ converges)

Q: Why does the Divergence Test work?

Want to show:

$$\underset{\lim_{k\to\infty} a_k \neq 0}{A} \implies \underset{\sum_{k=1}^{\infty} a_k \text{ diverges}}{B}$$

It is equivalent to show

$$\underset{\lim_{k\to\infty} a_k = 0}{\text{not } A} \impliedby \underset{\sum_{k=1}^{\infty} a_k \text{ converges}}{\text{not } B}$$

Assume $\sum_{k=1}^{\infty} a_k$ converges

that S_n (partial sums) have a finite limit

$\lim_{n\to\infty} S_n = L$ $\quad$ (L is a finite number)

$(S_n = a_1 + a_2 + \dots + a_n)$

($S_n = \ldots$

Want: $\lim_{k\to\infty} a_k = \lim_{k\to\infty} \{ S_k - S_{k-1} \}$

$$\lim_{k\to\infty} a_k = \lim_{k\to\infty} \{ (\cancel{a_1} + \cancel{a_2} + \dots + \cancel{a_{k-1}} + a_k) - (\cancel{a_1} + \cancel{a_2} + \dots + \cancel{a_{k-1}}) \}$$

$$\lim_{k\to\infty} a_k = \lim_{k\to\infty} \{ S_k - S_{k-1} \}$$

$$= \lim_{k\to\infty} S_k - \lim_{k\to\infty} S_{k-1}$$

$$= L - L = 0$$

$$\boxed{\begin{array}{l} S_n \to L \\ \sum a_k \to L \\ a_k \to 0 \neq L \end{array}}$$

$$\lim_{k\to\infty} a_k = 0$$

Review for Exam 2: Lesson 10 - 18

Lesson 10: Integration by Parts:

$$\int u\,dv = u \cdot v - \int v\,du$$

Ex: $\int \ln x\,dx$ $\to$ $u = \ln x$, $dv = dx$, $du = \frac{1}{x}dx$, $V = x$

$$= x \ln x - \int \cancel{x} \cdot \frac{1}{\cancel{x}}\,dx = x \ln x - x + C$$

Ex: $\int x \cos(x)\,dx$ $\to$ $u = x$, $dv = \cos(x)\,dx$, $du = dx$, $V = \sin(x)$

$$= x \sin(x) + \cos(x) + C$$

Lesson 11: $\int \cos^m(x) \sin^n(x)\,dx = ?$

m n method

odd		$u = \sin(x)$ $du = \cos(x)dx$ $\cos^2(x) + \sin^2(x) = 1$
	odd	$u = \cos(x)$ $du = -\sin(x)dx$
even	even	Half angle formulas $\cos^2(x) = \frac{1+\cos(2x)}{2}$ $\sin^2(x) = \frac{1-\cos(2x)}{2}$

<u>Lesson 12:</u> $\int \tan^m(x)\sec^n(x)dx$

m	n	method
	even	$u = \tan(x)$ $du = \sec^2(x)dx$ $\tan^2(x) + 1 = \sec^2(x)$
odd	odd	$u = \sec(x)$ $du = \tan(x)\sec(x)dx$
even	odd	Integration by parts $\tan^2(x) + 1 = \sec^2(x)$

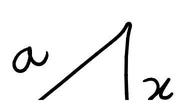
<u>lesson 13-15:</u> Trig substitutions

$\boxed{1}$ $\sqrt{a^2 - x^2} \longrightarrow$ $x = a\sin\theta$, $dx = a\cos\theta\, d\theta$

$\downarrow$

$\sqrt{a^2(1-\sin^2\theta)} = \sqrt{a^2\cos^2\theta} = a\cos\theta$

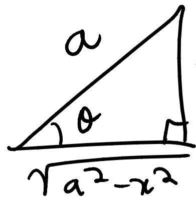
Refrence



$x = a\sin\theta$

$\sin\theta = \frac{x}{a}$

Refrence Triangle



$x = a\sin\theta$

$\sin\theta = \dfrac{x}{a}$

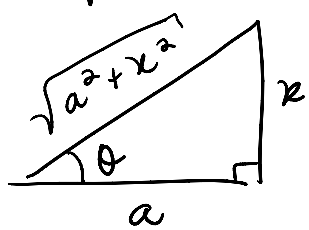
$\boxed{2}$ $\sqrt{a^2+x^2} \longrightarrow x = a\tan\theta$

$dx = a\sec^2\theta\, d\theta$

$\downarrow$

$\sqrt{a^2(1+\tan^2\theta)} = \sqrt{a^2\sec^2\theta} = a\sec\theta$

Reference Triangle



$x = a\tan\theta$

$\tan\theta = \dfrac{x}{x}$

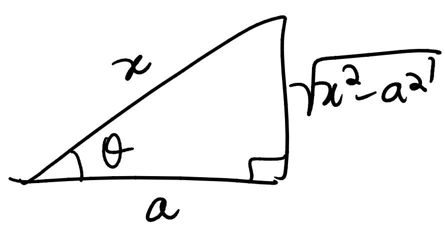
$\boxed{3}$ $\sqrt{x^2-a^2} \longrightarrow x = a\sec\theta$

$dx = a\tan\theta\sec\theta\, d\theta$

$\downarrow$

$\sqrt{a^2(\sec^2\theta-1)} = \sqrt{a^2\tan^2\theta} = a\tan\theta$

Reference Triangle



$x = a\sec\theta$

$\sec\theta = \dfrac{x}{a}$

★ May need to complete square first

i.e. $\sqrt{x^2+bx+c} \xleftrightarrow{\text{Rewrite}} \sqrt{(x-d)^2 \pm a^2}$