Lesson 32 - MA 166 LEC 200

12.2: Polar Coordinates (Basics)

Announcements:

- MyLab Math outage this morning
- will give an extension on HW 31 and Quiz 10

Warm Up:

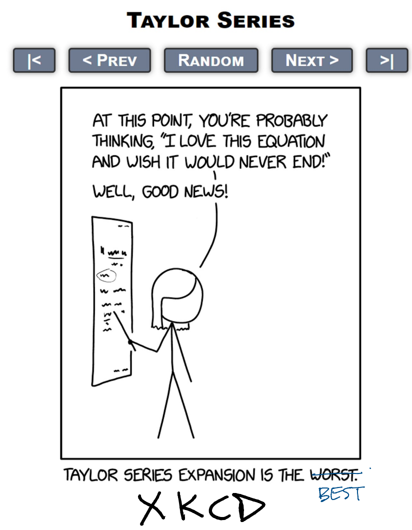
Find the power series of $\int e^{-x^2}\,dx$

(a) $C + \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{x^{2k+1}}{(2k+1)}$

(b) $C + \sum_{k=0}^{\infty} \frac{1}{k!} \frac{x^{k+3}}{(k+3)}$

(c) $C + \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)} x^{2k+1}$

(d) $C + \sum_{k=0}^{\infty} \frac{1}{k!} \frac{x^{k+1}}{(k+1)}$



$$e^y = \sum_{k=0}^{\infty} \frac{y^k}{k!} \qquad \text{let } y = -x^2$$

$$\int e^{-x^2}\,dx = \int \sum_{k=0}^{\infty} \frac{(-x^2)^k}{k!}\,dx = \int \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{k!}\,dx$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \int x^{2k}\,dx = \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{x^{2k+1}}{2k+1} \right) + C$$

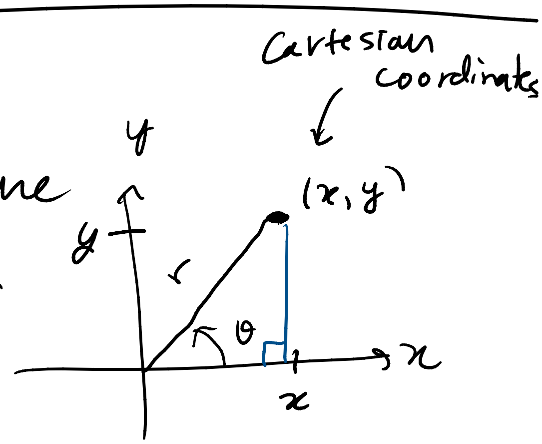
I. Polar Coordinates:

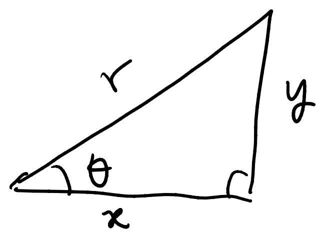
Given a point in the x-y plane

Can also write in polar coordinates

(r, θ) – polar

(x, y) – cartesian





Pythagorean Thm:

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

Trig:

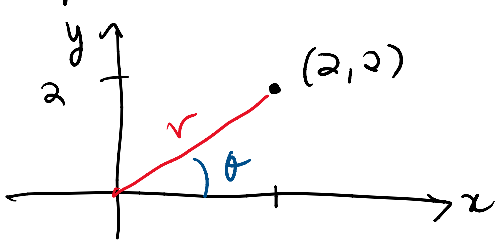
$$\tan\theta = \frac{y}{x}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Converting between coordinate systems

$$\begin{cases} r = \sqrt{x^2+y^2} \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) \end{cases} \qquad \begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

__Ex__ (1): Given $(x,y) = (2,2)$ in Cartesian plot and write this point in polar



$$r = \sqrt{x^2+y^2} = \sqrt{2^2+2^2}$$

$$\boxed{r = 2\sqrt{2}}$$

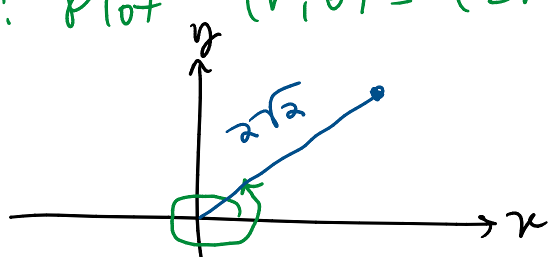
$$\theta = \tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1)$$

$$\boxed{\theta = \frac{\pi}{4}}$$

$$\boxed{(r,\theta) = \left(2\sqrt{2}, \frac{\pi}{4}\right)}$$

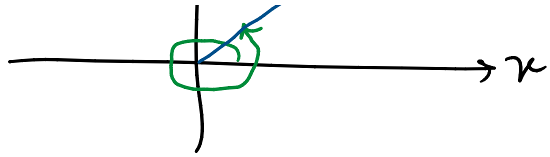
__Q__: Plot $(r,\theta) = \left(2\sqrt{2}, \frac{9\pi}{4}\right)$

$$\frac{9\pi}{4} = 2\pi + \frac{\pi}{4}$$



$\left(2\sqrt{2}, \frac{9\pi}{4}\right)$ and

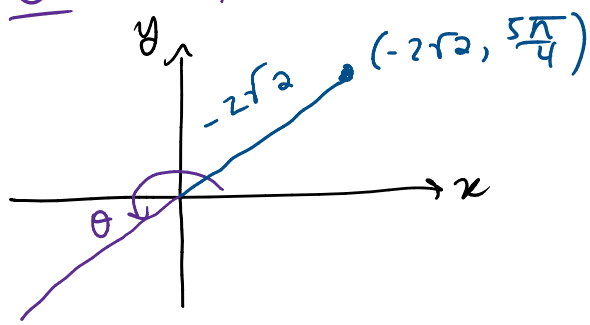
$\left(2\sqrt{2}, \frac{\pi}{4}\right)$ represent



$(2\sqrt{2}, \frac{\pi}{4})$ represent the same point.

NOTE: radius r can also be negative

Q: $(r, \theta) = (-2\sqrt{2}, \frac{5\pi}{4})$



same point again

$= (2\sqrt{2}, \frac{\pi}{4})$

$= (2\sqrt{2}, \frac{9\pi}{4})$

$= (-2\sqrt{2}, \frac{5\pi}{4})$

Equivalent points in Polar Coordinates:

$$(r, \theta) \equiv (r, \theta + 2\pi) \equiv (-r, \theta + \pi)$$

all represent the same point

HOTSEAT: Which of the following points are equivalent:

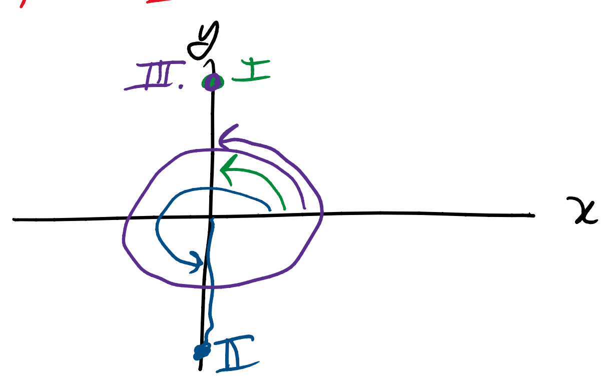
I. $(2, \frac{\pi}{2})$ II $(2, \frac{3\pi}{2})$ III $(2, \frac{5\pi}{2})$

(a) I, II, and III

(b) I and III only

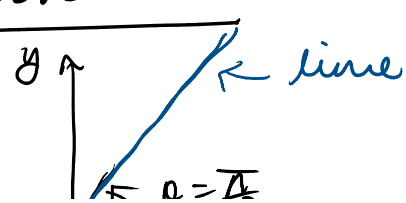
(c) I and II only

(d) II and III only



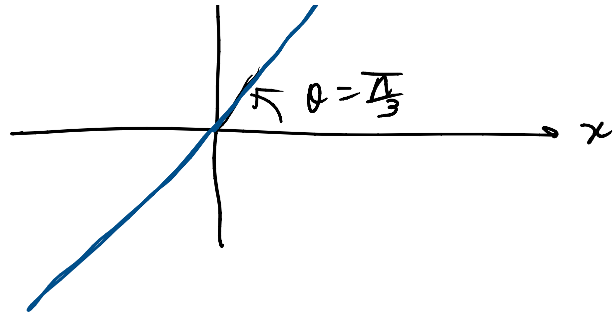
II. Basic Curves in Polar Coordinates:

① $\theta = \frac{\pi}{3}$



① $\theta = \frac{\pi}{3}$

is a line



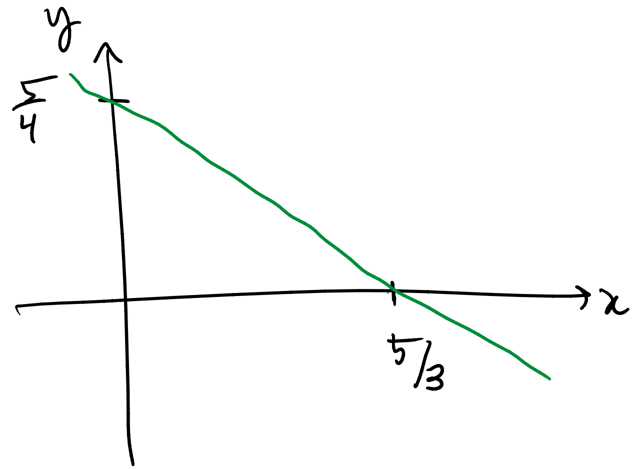
② $r = \dfrac{5}{3\cos\theta + 4\sin\theta}$

$r(3\cos\theta + 4\sin\theta) = 5$

$3\underbrace{r\cos\theta}_{x} + 4\underbrace{r\sin\theta}_{y} = 5$

$3x + 4y = 5$

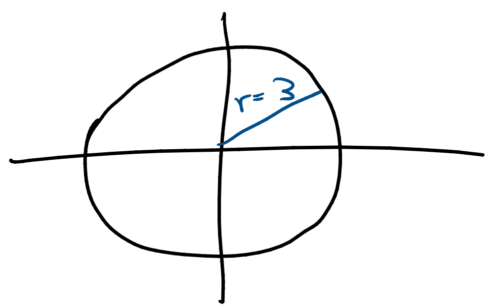
$y = -\frac{3}{4}x + \frac{5}{4}$ $\boxed{\text{line}}$



③ $r = 3$

$\boxed{\text{circle}}$

center $(0,0)$



④ $[r = 2\sin\theta]\, r$

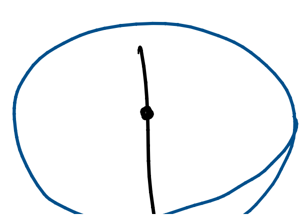
$\underbrace{r^2}_{x^2+y^2} = 2\underbrace{r\sin\theta}_{y}$

$x^2 + y^2 = 2y$

circle

radius $r = 1$

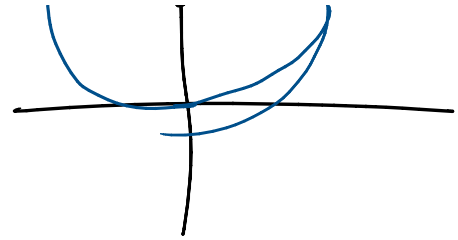
center $(0,1)$



$$x^2 + y^2 = 2y$$

$$x^2 + y^2 - 2y + 1 = 0 + 1$$

$$x^2 + (y-1)^2 = 1$$



HOTSEAT: The equation $r = 10\cos\theta + 24\sin\theta$ represents a circle. Find the radius r and center (x,y) of the circle

$\boxed{\text{(a) } r = 13 \text{ and } (x,y) = (5,12)}$

(b) $r = 13$ and $(x,y) = (10,24)$

(c) $r = 169$ and $(x,y) = (10,12)$

$$r^2 = 10r\cos\theta + 24r\sin\theta$$

$$(x^2 - 10x + 5^2) + (y^2 - 24y + 12^2) = 169$$

$$(x-5)^2 + (y-12)^2 = 13^2$$

center $(5,12)$ radius is 13

Equation of a circle:

$$r = 2a\cos\theta + 2b\sin\theta$$

has radius $\sqrt{a^2+b^2}$

center (a,b)

III Cartesian to Polar:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

Ex: Convert the equation to polar coordinates

$$y = x^2$$

$$r\sin\theta = (r\cos\theta)^2 = r^2\cos^2\theta$$

Want: $r = f(\theta)$

$$\frac{r^{\cancel{2}}\,\cancel{\cos^2\theta}}{\cancel{r}\,\cancel{\cos^2\theta}} = \frac{\cancel{r}\sin\theta}{\cancel{r}\cos^2\theta}$$

$$r = \frac{\sin\theta}{\cos^2\theta} = \left(\frac{\sin\theta}{\cos\theta}\right)\cdot\frac{1}{\cos\theta}$$

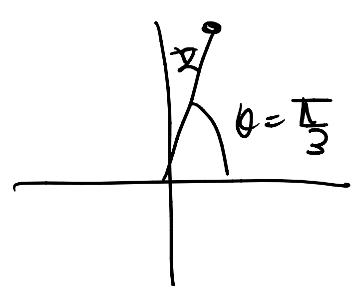
$$\boxed{r = \tan\theta\sec\theta}$$

Ex: Given $(x, y) = (1, \sqrt{3})$ convert to polar (r, θ)

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$

$$\boxed{(r, \theta) = \left(2, \frac{\pi}{3}\right)}$$



$$\boxed{(r, \theta) = \left(2, \frac{\pi}{3}\right)}$$