

12.2 & 12.3:
Calculus in Polar
Coordinates - Part 1

Announcements:

Quiz 10 due Fri 4/15 @ 11:59pm
HW 32 due Sun 4/17 @ 11:59pm
(b/c of the mylab math outage)

★ Warm Up: Convert the following equation into
Cartesian Coordinates: $r = 3 \csc \theta$

(a) $y = 3x$

$r = 3 \csc \theta$

(b) $y = 3$

$r = \frac{3}{\sin \theta}$

(c) $x = \frac{1}{3}$

(d) $y = 3+x$

$y = r \sin \theta = 3$ $y = 3$

EXAM 3 - Wed Apr 20 @ 6:30-7:30pm in ELLT

→ same* assigned seats
(* a few changed - you should have received)
an email w/ a notification

Material: Lessons 19 - 31

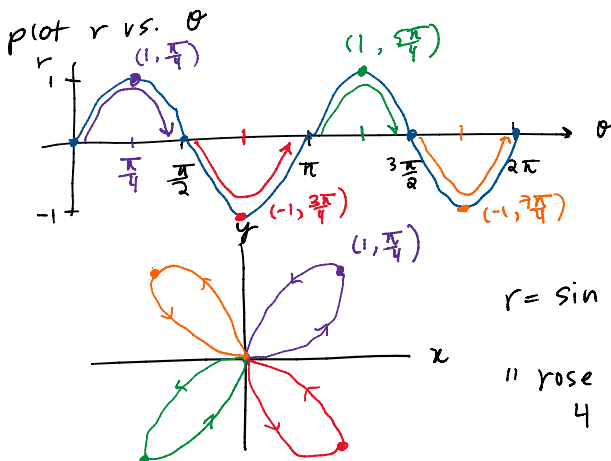
Chapters 10.1 - 10.8, 11.1 - 11.4

- sequences + series
 - convergence tests
- Power series
 - Taylor polynomials
 - Taylor + Maclaurin Series
 - Interval + Radius of convergence

I. Graphing with Polar Coordinates

Last class: - lines
- circles

EX(1): $r = \sin(2\theta)$ → doesn't easily convert to cartesian

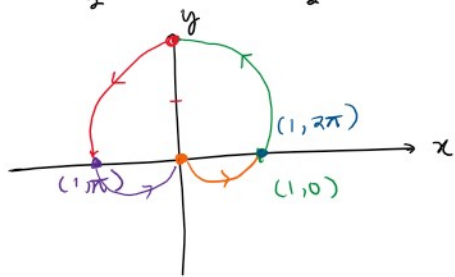
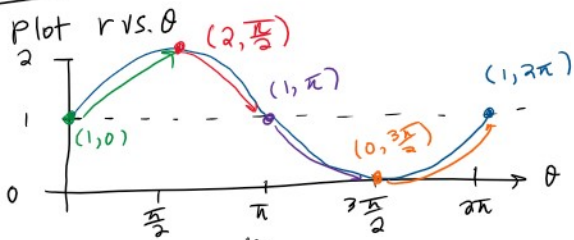


$r = \sin(2\theta)$

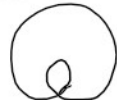
"rose with 4 leaves"

EX(2): Plot $r = 1 + \sin \theta$

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Cardioid



$$r = 1 + 2 \sin \theta$$

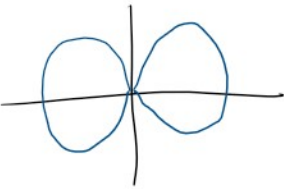
$$r = 1 + \frac{1}{2} \sin \theta$$



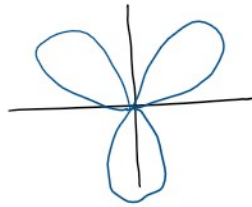
HOTSEAT:

What does the graph of $r^2 = 9 \cos \theta$ look like?

(a)

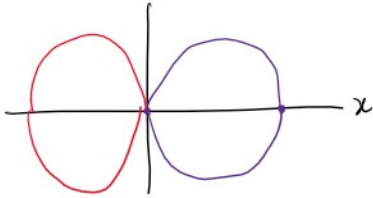
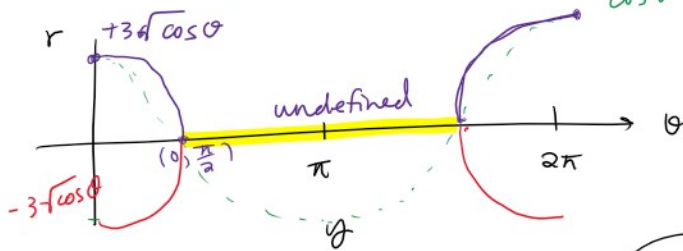


(b)



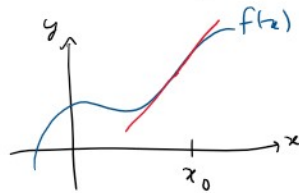
$$r = \pm 3 \sqrt{\cos \theta}$$

plot r vs θ
 $\cos \theta$



Lemniscate

rose with 2 petals



II. Slope of Tangent Lines:

In cartesian coordinates

$y = f(x)$ find the slope at $x = x_0$

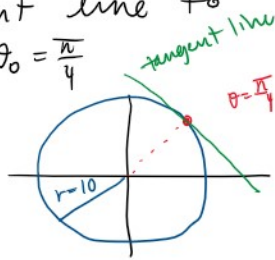
$$\text{slope} = \left. \frac{dy}{dx} \right|_{x=x_0} = f'(x_0)$$

In polar coords:
 $r = f(\theta)$ find the slope at θ_0

Recall: $x = r \cos \theta = f(\theta) \cos \theta$
 $y = r \sin \theta = f(\theta) \sin \theta$

slope: $\left. \frac{dy}{dx} \right|_{\theta=\theta_0} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta} [f(\theta) \sin \theta]}{\frac{d}{d\theta} [f(\theta) \cos \theta]}$
 $= \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta} \Big|_{\theta=\theta_0}$

EX: Find the slope of the tangent line to the circle $r = f(\theta) = 10$ at $\theta_0 = \frac{\pi}{4}$



slope $\frac{dy}{dx} = \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta} \Big|_{\theta=\frac{\pi}{4}}$

$= \frac{10 \cos \theta}{-10 \sin \theta} \Big|_{\theta=\frac{\pi}{4}} = -\cot \theta \Big|_{\theta=\frac{\pi}{4}}$

$f'(\theta) = 0$

Slope = -1

HOTSEAT: What is the slope of $r=10$ at $\theta_0 = \pi$?

slope = $-\cot \theta \Big|_{\theta=\pi} = \frac{-\cos(\pi)}{\sin(\pi)} = \frac{-1}{0} \rightarrow \text{undefined}$

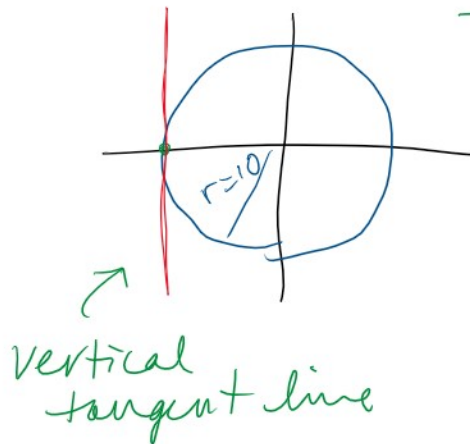
(a) 1

(b) 0

(c) undefined

(d) $\frac{1}{2}$

(e) $\frac{\sqrt{3}}{2}$



Ex: Given $r = f(\theta)$

Find the θ at which the graph
has horizontal tangent lines \rightarrow slope = 0
vertical tangent lines
 \rightarrow slope is undefined