12.2 & 12.3:
Calculus in Polar Coordinates – Part 1

Announcements:
- Quiz 10 due Fri 4/15 @ 11:59pm
- HW 32 due Sun 4/17 @ 11:59pm (b/c of the MyLab Math outage)

☆ Warm Up: Convert the following equation into Cartesian Coordinates: $r = 3\csc\theta$

(a) $y = 3x$

(b) $y = 3$

(c) $x = \frac{1}{3}$

(d) $y = 3 + x$

$r = \frac{3}{\sin\theta}$

$y = r\sin\theta = 3$

$y = 3$

EXAM 3 – Wed Apr 20 @ 6:30-7:30pm in ELLT
→ same* assigned seats
(* a few changed – you should have received an email w/ a notification)

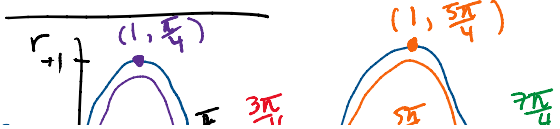
Material: Lessons 19 – 31
Chapters 10.1 – 10.8, 11.1 – 11.4
- sequences & series
  - convergence tests
- Power series
  - Taylor polynomials
  - Taylor & Maclaurin series
  - Interval & Radius of convergence

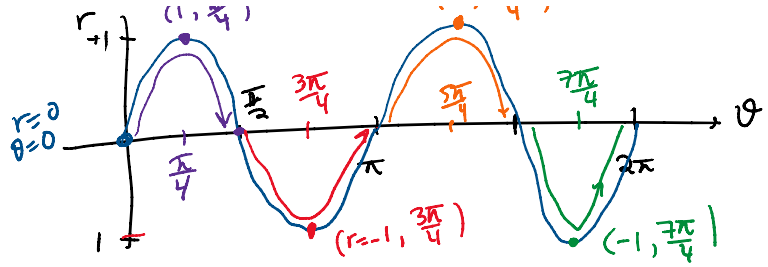
I. Graphing in Polar Coordinates:

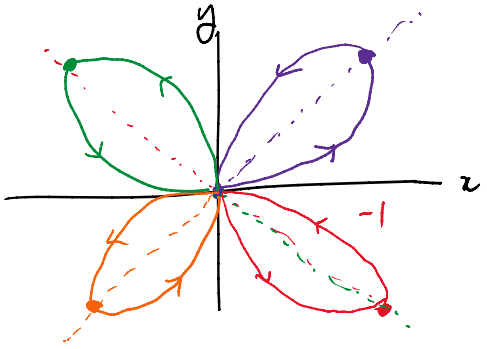
Last class:
- lines
- circles

Ex(1): $r = \sin(2\theta)$ → doesn't easily convert to cartesian

Plot r vs. θ:



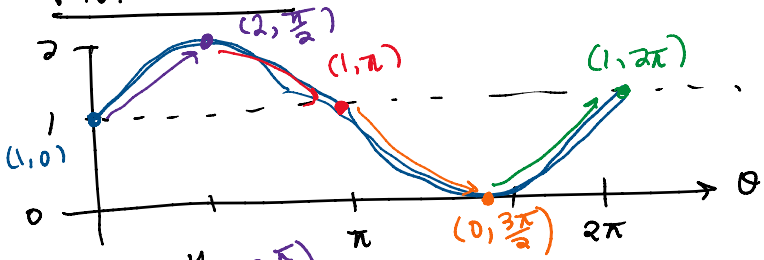


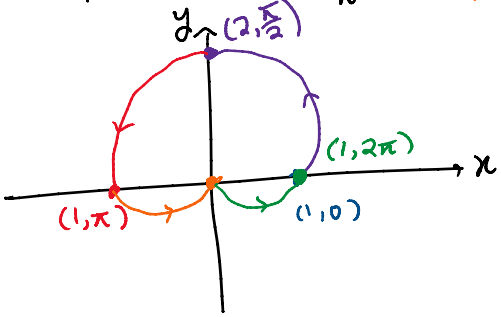


"rose with 4 leaves"

EX(2): Plot $r = 1 + \sin\theta$

Plot r vs θ:

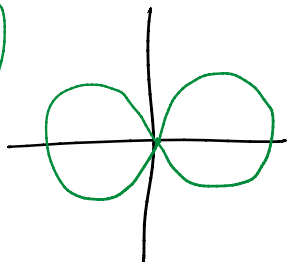




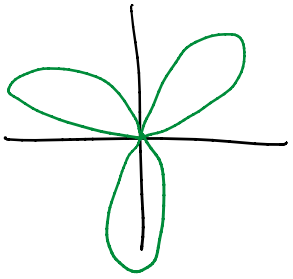
cardioid

HOTSEAT: What does the graph of $r^2 = 9\cos\theta$ look like

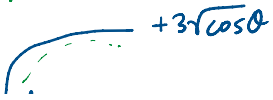
(a)

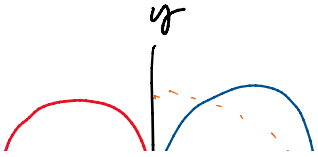


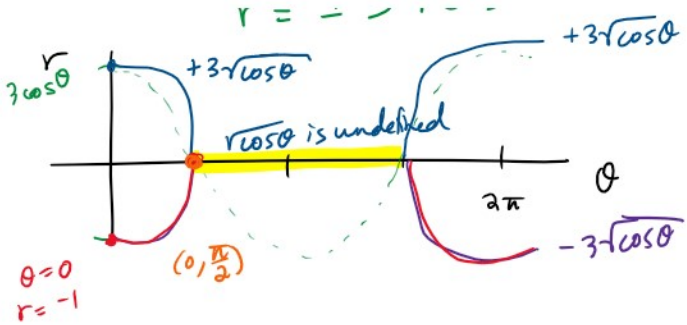
(b)

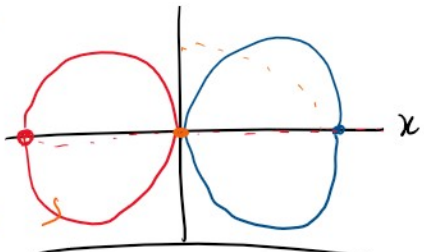


$r = \pm 3\sqrt{\cos\theta}$

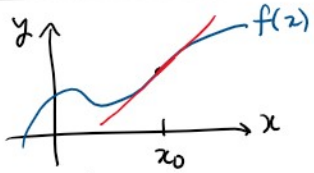








Lemniscate



II. Slope of Tangent Lines:

In Cartesian:

$y = f(x)$ find the slope at $x = x_0$

$$\text{slope} = \left.\frac{dy}{dx}\right|_{x=x_0} = f'(x_0)$$

In Polar coordinates:

$r = f(\theta)$ find the slope at θ_0

Recall: $x = r\cos\theta = f(\theta)\cos\theta$

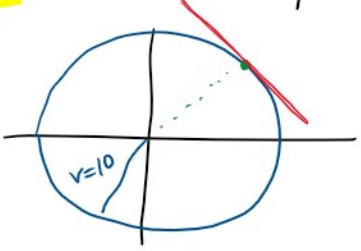
$y = r\sin\theta = f(\theta)\sin\theta$

$$\text{slope: } \left.\frac{dy}{dx}\right|_{\theta=\theta_0} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta}[f(\theta)\sin\theta]}{\frac{d}{d\theta}[f(\theta)\cos\theta]}$$

$$= \left.\frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}\right|_{\theta=\theta_0}$$

Ex (3): Find the slope of the tangent line to the circle $r = f(\theta) = 10$ at $\theta_0 = \frac{\pi}{4}$

$f'(\theta) = 0$



$$\text{slope} = \frac{dy}{dx}$$

$$= \left.\frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{\cos\theta - f(\theta)\sin\theta}\right|_{\theta=\frac{\pi}{4}}$$

$$= \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}\Bigg|_{\theta=\frac{\pi}{4}}$$

$$= \frac{10\cos\theta}{-10\sin\theta}\Bigg|_{\theta=\frac{\pi}{4}} = -\cot\left(\frac{\pi}{4}\right) = \boxed{-1 \quad = \text{slope}}$$

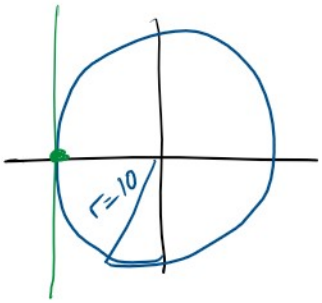
HOT SEAT: What is the slope of $r = 10$ at $\theta_0 = \pi$?

(a) 1
(b) 0
(c) undefined
(d) $\frac{1}{2}$
(e) $\frac{\sqrt{3}}{2}$

$$\text{slope} = -\cot(\theta)\Big|_{\theta=\pi}$$

$$= \frac{-\cos(\pi)}{\sin(\pi)} = \frac{+1}{0}$$

undefined



NOTE:

Horizontal tangent lines slope = 0

Vertical tan " slope = undefined