6.2: Regions Between Curves

Announcements:

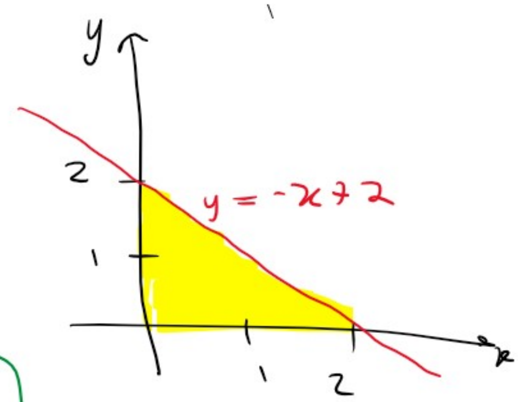
- Right-hand rule for vector cross product
- HW 2 problems – tech issues points will be returned

GOALS:

- find the area between 2 curves

☆ Warm Up:

What is the area A between the line $y = -x + 2$ and the x-axis for $0 \le x \le 2$



(a) $A = 3$ (b) $A = 4$ $\boxed{(c)\ A = 2}$

$$A = \int_a^b f(x)\,dx = \int_0^2 (-x+2)\,dx = \left[-\frac{x^2}{2} + 2x\right]_0^2$$

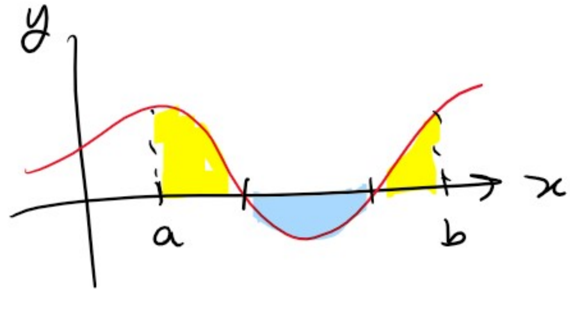
$$= -\frac{4}{2} + 4 - (0+0) = \boxed{2}$$

I. Definite Integrals: (sec 5.2 in text)

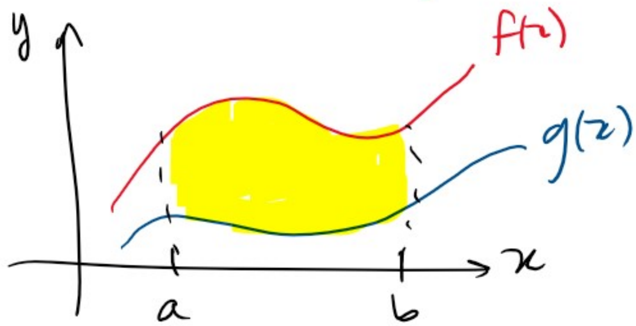
use definite integrals to calculate the net area under a curve $f(x)$

$$A = \int_a^b f(x)\,dx$$

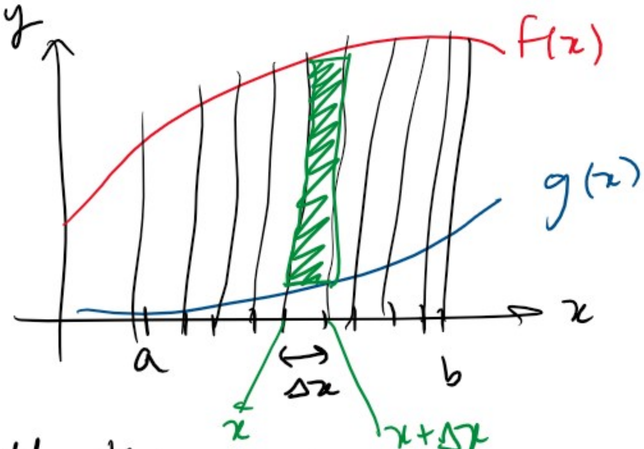
= yellow region − blue region



Q: What if we replace the x-axis with another curve $g(x)$



Riemann sum



area of box

$= [f(x) - g(x)]\Delta x$

sum up over all the boxes

$$A \approx \sum_{k=1}^{n} [f(x_k) - g(x_k)]\Delta x$$

limit as $n \to \infty$

$$\boxed{A = \int_a^b [f(x) - g(x)]\,dx}$$

Def: The area of the region bounded by the graphs of f and g on [a,b]

f, g are continuous, $f(x) \geq g(x)$

EX: Find the area bounded by $f(x) = 5 - x^2$ and $g(x) = x^2 - 3$

1. Find the [a,b] → intersections pts of f and g

intersect when: $\quad f(x) = g(x)$

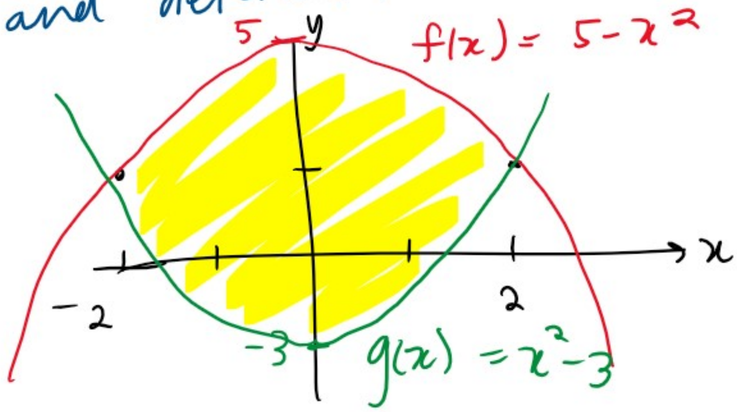
$$5 - x^2 = x^2 - 3$$

$[-2, 2]$ $\qquad 8 = 2x^2$

$f(2) = 1 = g(2)$ $\qquad 4 = x^2 \qquad x = \pm 2$

$f(-2) = 1 = g(-2)$

2. Plot and determine which fcn is larger



$f(x) \geq g(x)$

3. Calculate area

$$A = \int_a^b [f(x) - g(x)]\,dx = \int_{-2}^{2} [5 - x^2 - (x^2 - 3)]\,dx$$

$$= \int_{-2}^{2} [8 - 2x^2]\,dx = \left[8x - \frac{2x^3}{3}\right]_{-2}^{2}$$

$$= \left[8(2) - \frac{2(2)^3}{3} - \left(8(-2) - \frac{2(-2)^3}{3}\right)\right]$$

$$= \left[16 - \frac{16}{3} + 16 - \frac{16}{3}\right] = 32 - \frac{32}{3} = 32$$

$$\boxed{A = \frac{64}{3}}$$

<u>Example</u>: Compound Region

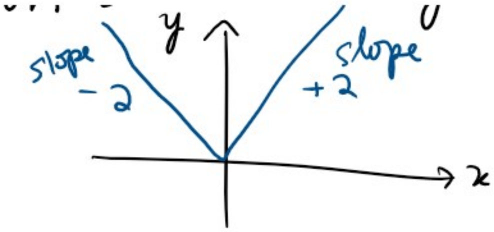
Find the area of the region bounded by

$\ldots = -x^2 + 3x + 6$

Find the area of the region

$f(x) = -x^2 + 3x + 6$

$g(x) = |2x|$



1. Find intersection points:

$\boxed{\text{if } x > 0}$ $\quad -x^2 + 3x + 6 = |2x| = 2x$

$-x^2 + x + 6 = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

roots: $x = +3, -2$

$x = +3$

$y = |2x| = 6$

$(3, 6)$

$\boxed{\text{if } x < 0}$ $\quad -x^2 + 3x + 6 = |2x| = -2x$

$-x^2 + 5x + 6 = 0$

$x^2 - 5x - 6 = 0$

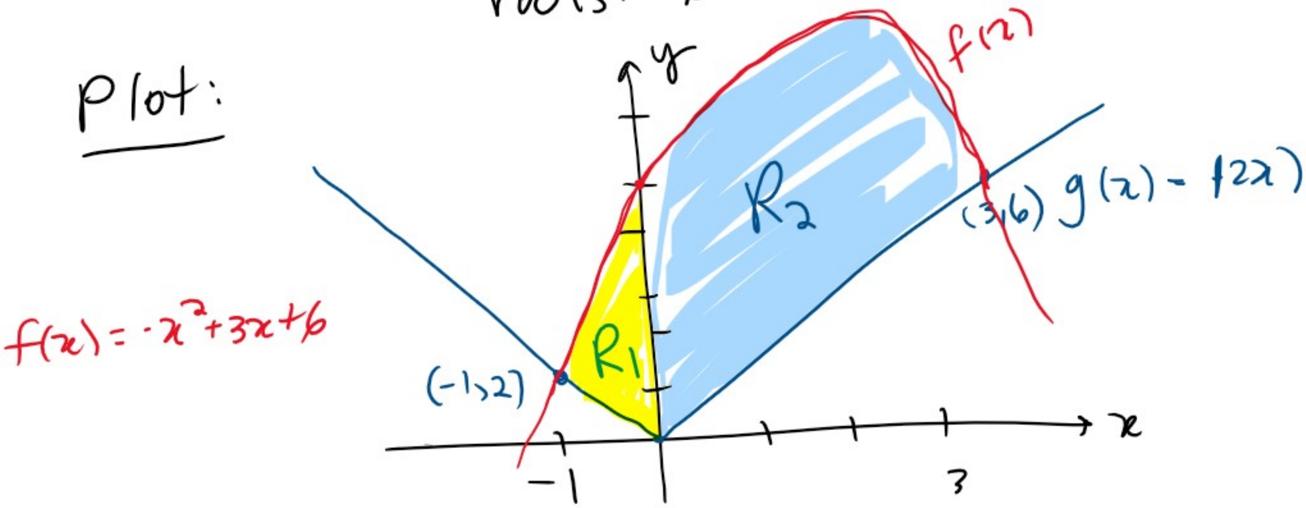
$(x-6)(x+1) = 0$

roots: $x = +6, -1$

$x = -1$

$y = |2x| = 2$

Plot:



$A = R_1 + R_2$

(a) $\int_{-1}^{0} [-x^2 + 3x + 6 - 2x]\,dx + \int_0^3 [-x^2 + 3x + 6 - 2x]\,dx$

(a) $\int_{-1}^{0} [-x^2 + 3x + 6 - 2x]\,dx + \int_{0} \ldots$

$$\boxed{(b)\ \int_{-1}^{0} [-x^2+3x+6+2x]\,dx + \int_{0}^{3} [-x^2+3x+6-2x]\,dx}$$

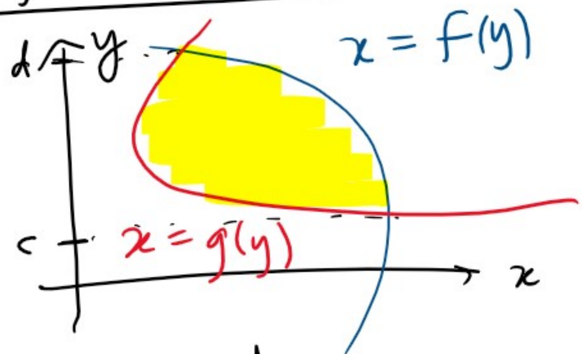
$$A = \int_a^b [f(x) - g(x)]\,dx$$

$$g(x) = |2x| = \begin{cases} 2x & \text{if } x > 0 \\ -2x & \text{if } x < 0 \end{cases}$$

$$R_1 = \int_{-1}^{0} [-x^2+3x+6 - (-2x)]\,dx$$

Evaluate integrals $\boxed{A = \frac{50}{3}}$

II. Integrals with respect to y :

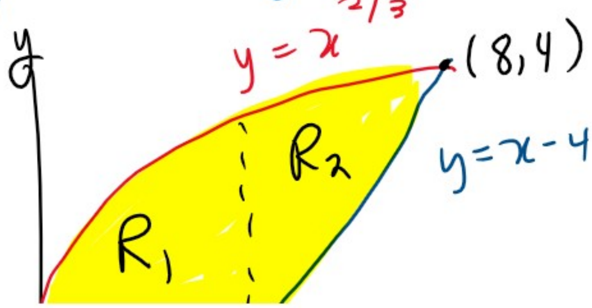


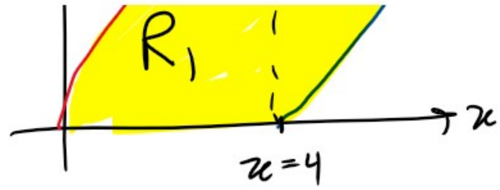
here $f(y) \geq g(y)$

then $A = \int_c^d [f(y) - g(y)]\,dy$

Ex: Find the area in the 1st quadrant bounded by $y = x^{2/3}$, $y = x - 4$

$\boxed{\text{2 way to solve}}$



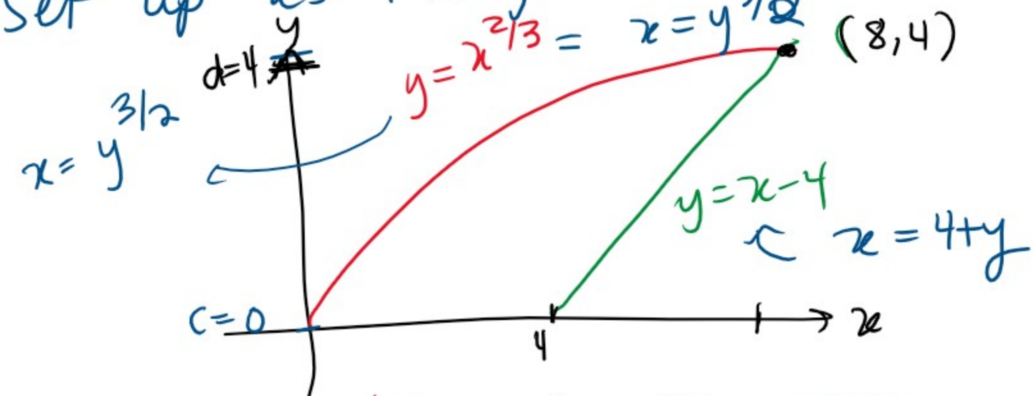


compound region:

$$A = \underbrace{\int_0^4 \left[x^{2/3} - 0\right] dx}_{R_1} + \underbrace{\int_4^8 \left[x^{2/3} - (x-4)\right] dx}_{R_2}$$

$\boxed{\text{Method 2}}$ Set up as integral in y

$y = x^{2/3}$
$x = y^{3/2}$



POLL: Set up the equations for the area as integral in y

$$\boxed{(a) \quad \int_0^4 \left[y+4 - y^{3/2}\right] dy}$$

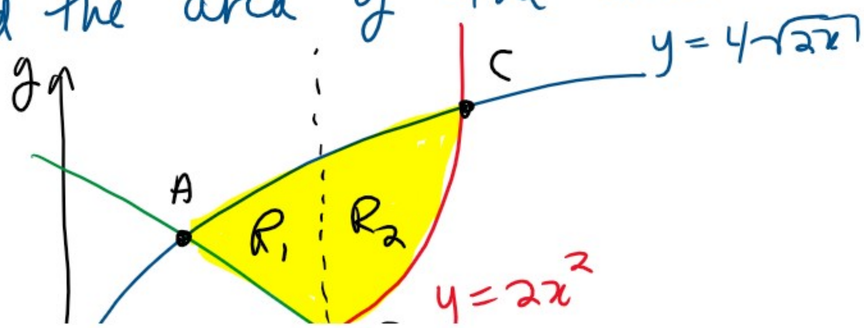
$$(b) \quad \int_0^8 \left[y^{3/2} - (y+4)\right] dy$$

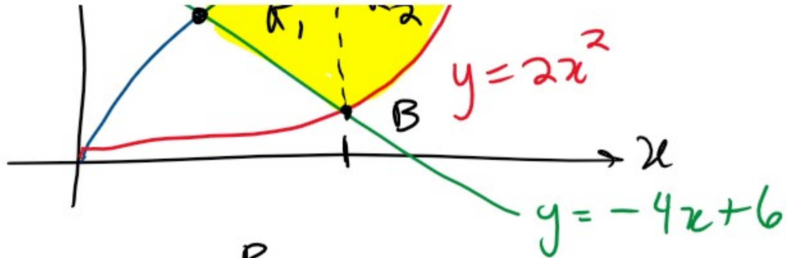
$4+y \geq y^{3/2}$

$$\int_0^4 \left[4+y - y^{3/2}\right] dy$$

Ex: Complicated Region

Find the area of the shaded region





$$R_1 = \int_A^B \left[4\sqrt{2x} - (-4x+6) \right] dx$$

$$R_2 = \int_B^C \left[4\sqrt{2x} - 2x^2 \right] dx \qquad A = R_1 + R_2$$