Lesson 5 - MA 166 LEC 286

6.2: Regions Between Curves

Announcements:

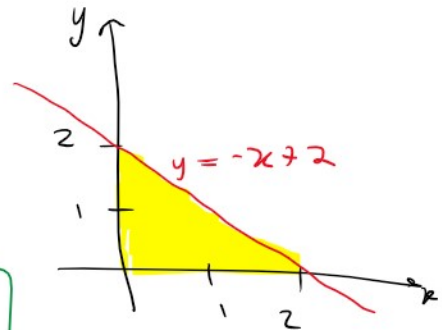
- Right-hand rule for vector cross product
- HW 2 problems – tech issues
  points will be returned

GOALS:

- find the area between 2 curves

Warm up:

What is the area A between the line $y = -x + 2$ and the x-axis for $0 \le x \le 2$



(a) $A = 3$ (b) $A = 4$ (c) $\boxed{A = 2}$

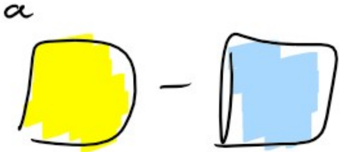
$$A = \int_a^b f(x)\,dx = \int_0^2 [-x+2]\,dx = \left[-\frac{x^2}{2} + 2x\right]_0^2$$

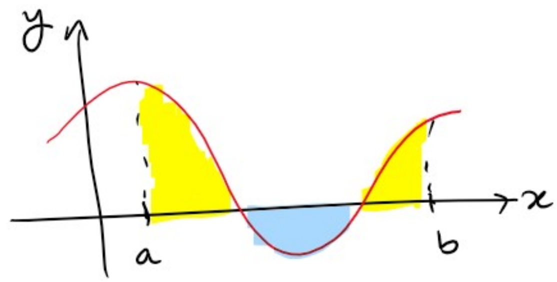
$$= -\frac{4}{2} + 4 \Rightarrow \boxed{A = 2}$$

I. Definite Integrals: (sec 5.2 in text)

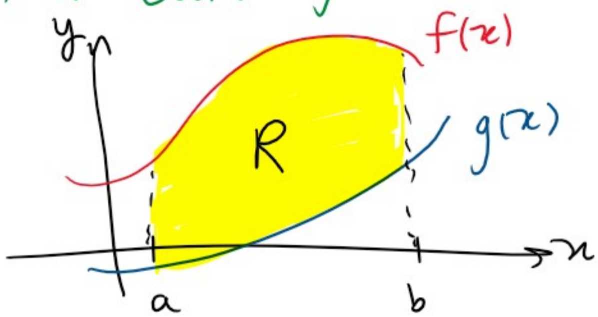
to calculate the net area under a curve $f(x)$

$$A = \int_a^b f(x)\,dx$$

$=$ 

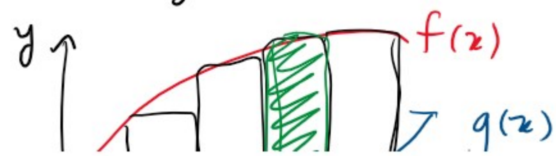


Q: What if we replace the x-axis with another curve $g(x)$?



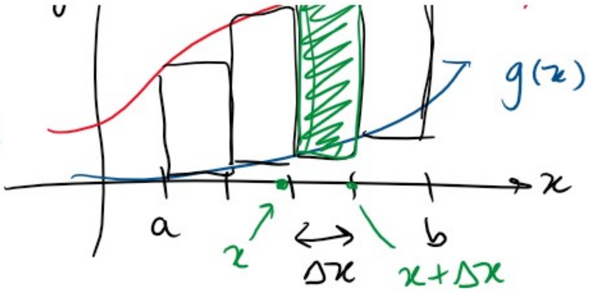
find area of region R

Riemann Sum:



Riemann Sum:

area of box $= [f(x)-g(x)]\Delta x$



$$A \approx \sum_{k=1}^{n} [f(x_k) - g(x_k)]\Delta x$$

limit as $n \to \infty$

$$\boxed{A = \int_a^b [f(x) - g(x)]\,dx}$$

Def: The area of the region bounded by the graphs of f and g on $[a,b]$

need f, g continuous and $f(x) \geq g(x)$

Ex: Find the area bounded by

$f(x) = 5 - x^2 \qquad g(x) = x^2 - 3$

1. Find $[a,b] \longrightarrow$ intersection points

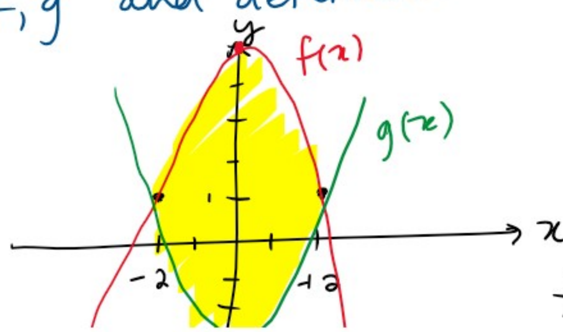
intersect when:

$$f(x) = g(x)$$
$$5 - x^2 = x^2 - 3$$
$$8 = 2x^2$$
$$4 = x^2 \qquad \boxed{x = \pm 2}$$

when $x = +2$
$y = f(2) = 5 - 4 = 1$
$x = -2$
$y = f(-2) = 5 - 4 = 1$

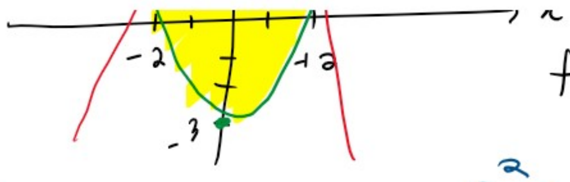
$(-2, 1)$ and $(+2, 1)$

2. Plot f, g and determine which is larger



$f(x) = 5 - x^2$
$g(x) = x^2 - 3$

$f(x) \geq g(x)$



$f(x) \geq g(x)$

3. $$A = \int_a^b [f(x) - g(x)]\,dx = \int_{-2}^{2} [5 - x^2 - (x^2 - 3)]\,dx$$

$$= \int_{-2}^{2} [8 - 2x^2]\,dx = \left[8x - \frac{2x^3}{3}\right]_{-2}^{2}$$

$$= \left[8 \cdot 2 - \frac{2(2)^3}{3}\right] - \left[8(-2) - \frac{2(-2)^3}{3}\right]$$

$$= \left[16 - \frac{16}{3}\right] + \left[+16 - \frac{16}{3}\right]$$

$$= 32 - \frac{32}{3} = \boxed{\frac{64}{3} = A}$$

__Example__: Compound Region

Find the area of the region bounded by

$$f(x) = -x^2 + 3x + 6$$

$$g(x) = |2x|$$

1. Find intersection points:

$$f(x) = g(x)$$

$$-x^2 + 3x + 6 = |2x| = \begin{cases} 2x & \text{if } x \geq 0 \\ -2x & \text{if } x < 0 \end{cases}$$

$\boxed{\text{if } x \geq 0}$

$$-x^2 + 3x + 6 = |2x| = +2x$$

$$-x^2 + x + 6 = 0$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

roots $x = +3$, -2

$$\boxed{\begin{array}{l} x = +3 \\ y = |2x| = 6 \\ (3, 6) \end{array}}$$

$\boxed{\text{if } x<0}$ $\quad -x^2+3x+6 = |2x| = -2x$

$-x^2+5x+6 = 0$

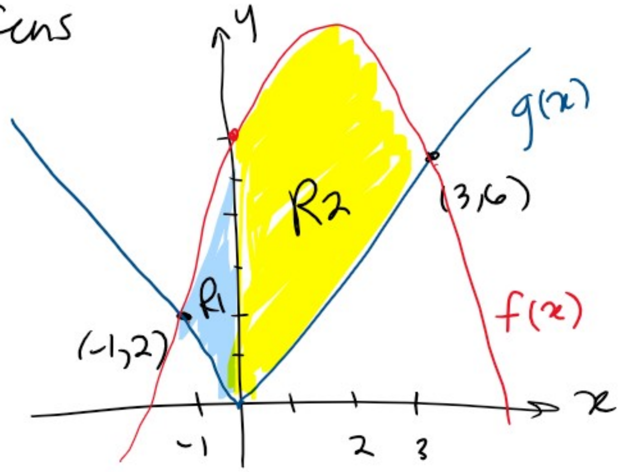
$x^2-5x-6 = 0$

$(x-6)(x+1) = 0$

roots: $x = +6, -1$

$\boxed{\begin{array}{l} x=-1 \\ y=|2x| = +2 \\ (-1,2) \end{array}}$

2. Plot fcns



$g(x) = |2x|$

$f(x) = -x^2+3x+6$

$g(x) = |2x| = \begin{cases} 2x & x \geq 0 \\ -2x & x<0 \end{cases}$

$A = R_1 + R_2$ POLL: Set up A as a sum of 2 integrals

(a) $\int_{-1}^{0}[-x^2+3x+6-2x]\,dx + \int_0^3[-x^2+3x+6-2x]\,dx$

$\boxed{\text{(b)} \quad \underbrace{\int_{-1}^{0}[-x^2+3x+6+2x]\,dx} + \int_0^3[-x^2+3x+6-2x]\,dx}$

$f(x) - g(x) = -x^2+3x+6-(-2x)$

Evaluates to $\boxed{A = \frac{50}{3}}$

II. Integrating with respect to y:

$A = \int_c^d [f(y)-g(y)]\,dy$

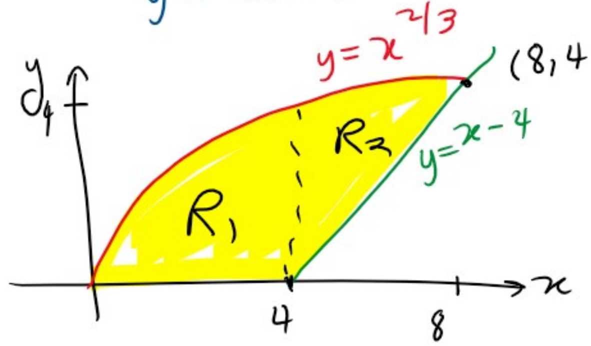
$f(y) \geq g(y)$

$f(y) \geq g(y)$

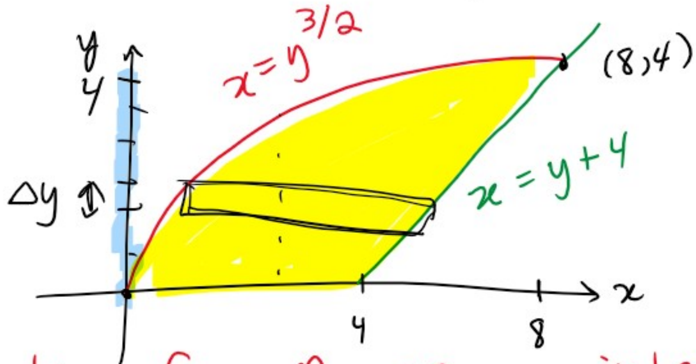
Ex: Find the area in the 1st quadrant bounded by
$y = x^{2/3}$
$y = x - 4$

Method 1:

$A = R_1 + R_2$



Method 2:



$(y+4-y^{2/3})$

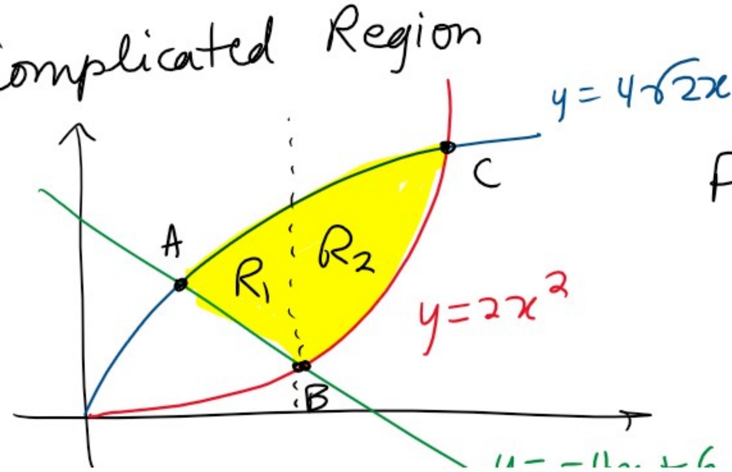
POLL:

Set up the equation for A as an integral with respect to y

$$\boxed{(a) \quad A = \int_0^4 \left[y+4 - y^{3/2} \right] dy}$$

$$(b) \quad A = \int_0^8 \left[y^{3/2} - (y+4) \right] dy$$

Ex: Complicated Region



Find area of the shaded region

$$A = R_1 + R_2$$

$$R_1 = \int_A^B \left[4\sqrt{2x} - (-4x+6) \right] dx$$

$$R_2 = \int_B^C \left[4\sqrt{2x} - 2x^2 \right] dx$$