Exam 1 Review - MA 166 LEC 200

EXAM 1 - Today @ 6:30pm - 7:30pm in ELLT

No Lecture on Friday Feb 11

Instructions:

- please be in your seat by 6:15pm (so we can start right at 6:30pm)
- Know your recitation section #

1:30 pm Lecture		
TA	Time & Location	Recitation Number
Connor Thompson	10:30 am in PHYS 110	0208
Connor Thompson	11:30 am in PHYS 110	0215
Connor Thompson	12:30 pm in PHYS 110	0222
Nikhil Mehra	4:30 pm in PHYS 202	0229
Nikhil Mehra	3:30 pm in PHYS 202	0236
Nikhil Mehra	2:30 pm in PHYS 202	0243
Sarah Anderson	10:30 am in PHYS 202	0250
Sarah Anderson	9:30 am in PHYS 202	0257
Sarah Anderson	11:30 am in PHYS 202	0264
Rishab Pandian	2:30 pm in PHYS 110	0271
Rishab Pandian	1:30pm in PHYS 110	0278
Rishab Pandian	3:30 pm in PHYS 110	0285

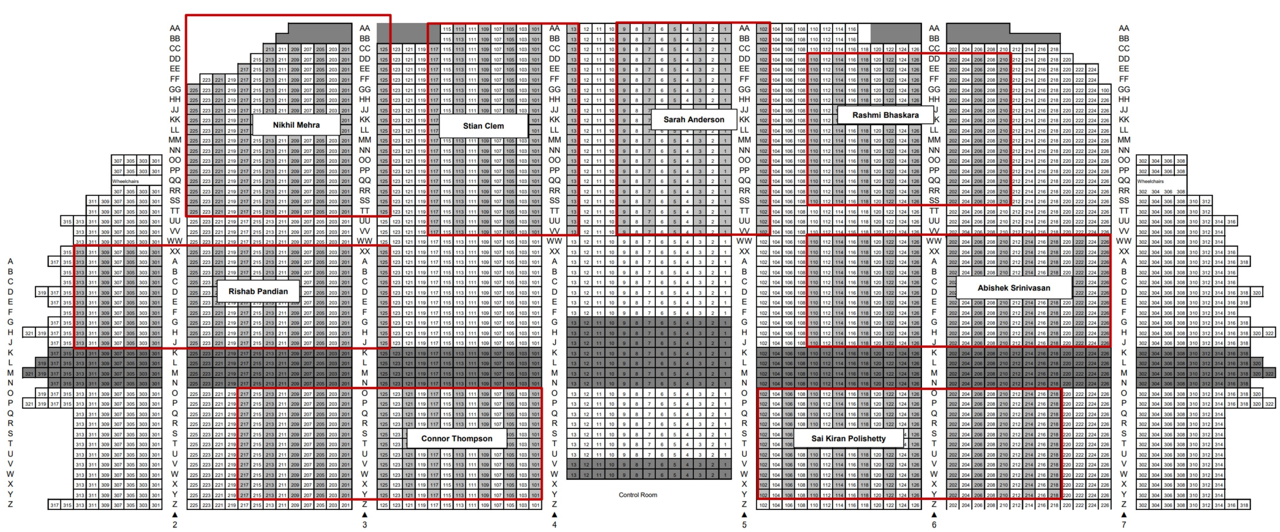
2:30 pm Lecture		
TA	Time & Location	Recitation Number
Saikiran Polishetty	9:30 am in HAMP 2101	0287
Saikiran Polishetty	7:30 am in HAMP 2101	0288
Saikiran Polishetty	8:30 am in HAMP 2102	0289
Stian Clem	8:30 am in PHYS 333	0290
Stian Clem	7:30 am in PHYS 333	0291
Stian Clem	9:30 am in PHYS 333	0292
Abishek Srinivasan	3:30 pm in PHYS 333	0293
Abishek Srinivasan	4:30 pm in PHYS 333	0294
Abishek Srinivasan	2:30 pm in PHYS 333	0295
Rashmi Bhaskara	9:30 am in PHYS 110	0296
Rashmi Bhaskara	7:30 am in PHYS 110	0297
Rashmi Bhaskara	8:30 am in PHYS 110	0298

- sit in your designated area:

MA 16600
Exam 1

Elliott Hall of Music
Purdue University
Main Floor
No seating Rows K, L, M, N - All Sections
No seating Rows G, H, J, K, L, M, N - Center Sections
3561/1700 Stations

Wednesday, Feb. 9, 2022
6:30 - 7:30 p.m.



Bring:
- #2 pencil
- PUID
- Recitation section #
- TAs name

☆ EXAM 1 Review: (Based on Hot Seat Poll)

1. Arc Length & Surface Area
2. Physical Applications
3. Shells vs. Washer vs. General Slicing Method
4. Lines, Planes, Spheres, & Vectors

I. Arc Length & Surface Area:

Arc length L of $f(x)$ on $[a,b]$

$$L = \int_a^b \sqrt{1 + \{f'(x)\}^2}\, dx$$

Surface area S of the solid of revolution:

Surface area S of the solid of revolution around x-axis:

$f(x)$ on $[a,b]$ revolved around x-axis:

$$S = \int_a^b 2\pi f(x) \sqrt{1 + \{f'(x)\}^2}\, dx$$

Ex: Find the arc length of

$f(x) = \ln|\cos(x)|$ on $[0, \frac{\pi}{4}]$

$f'(x) = \frac{1}{\cos(x)} \cdot -\sin(x) = -\tan(x)$

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \{-\tan(x)\}^2}\, dx$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2(x)}\, dx = \int_0^{\frac{\pi}{4}} \sqrt{\sec^2(x)}\, dx$$

$u = \sec(x) + \tan(x)$ …

$du = $

$\int \frac{du}{u}$

$$= \int_0^{\frac{\pi}{4}} \sec(x)\, dx = \ln|\sec(x) + \tan(x)| \Big]_0^{\frac{\pi}{4}}$$

$$= \ln\left|\frac{1}{\cos(\frac{\pi}{4})} + \overset{1}{\tan(\tfrac{\pi}{4})}\right| - \ln\left|\overset{1}{\frac{1}{\cos(0)}} + \overset{0}{\tan(0)}\right|$$

$\int \frac{du}{u}$

$$= \ln\left|\frac{1}{\cos(\frac{\pi}{4})} + \cancelto{1}{\tan\left(\frac{\pi}{4}\right)}\right| - \ln\left|\cancelto{1}{\frac{1}{\cos(0)}} + \cancelto{0}{\tan(0)}\right|$$

$$= \ln\left|\sqrt{2}+1\right| - \cancelto{0}{\ln|1|} = \boxed{\ln|1+\sqrt{2}|}$$

Practice: $f(x) = \frac{x^4}{4} + \frac{1}{8x^2}$ on $[1,2]$

Spring 2020 #11

11. Find the area of the northern cap of the sphere of radius 1 when the cap has height 1/2.

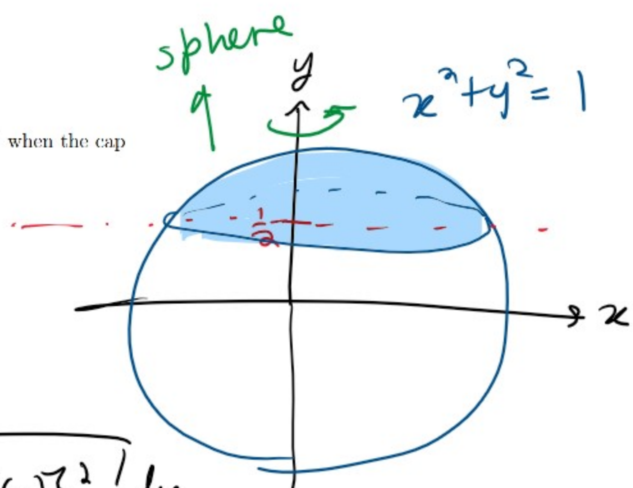
A. π

B. π^2

C. $\frac{4\pi}{3}$

D. $\frac{2\pi}{3}$

E. $\frac{\pi}{4}$



$$S = \int_{\frac{1}{2}}^{1} 2\pi f(y)\sqrt{1+\{f'(y)\}^2}\,dy$$

$$f(y) = \sqrt{1-y^2} \qquad f'(y) = \frac{1}{2}(1-y^2)^{-1/2}(-2y)$$

$$= \frac{-y}{\sqrt{1-y^2}}$$

$$= \int_{\frac{1}{2}}^{1} 2\pi\sqrt{1-y^2}\cdot\sqrt{1+\left(\frac{-y}{\sqrt{1-y^2}}\right)^2}\,dy$$

$$= \int_{\frac{1}{2}}^{1} 2\pi\sqrt{1-y^2}\sqrt{\frac{1-\cancel{y^2}}{1-y^2}+\frac{\cancel{y^2}}{1-y^2}}\,dy$$

$$= \int_{\frac{1}{2}}^{1} 2\pi\cancel{\sqrt{1-y^2}}\cdot\frac{1}{\cancel{\sqrt{1-y^2}}}\,dy = \int_{\frac{1}{2}}^{1} 2\pi\,dy$$

$$= \Big[2\pi y\Big]_{\frac{1}{2}}^{1} = 2\pi - \pi = \boxed{\pi}\ \boxed{A}$$

II. Physical Applications:

Spring — Hooke's Law $F = kx$

★ x is the displacement from equilibrium position (natural length)

$$W = \int_a^b kx\,dx$$

Spring 2018 #7

7. If the work required to stretch a spring 3 ft beyond natural length is 36 ft-lb, how much work is needed to stretch it 3 inches beyond natural length ?

A. $\frac{3}{8}$ ft-lb
B. 54 ft-lb
C. 36 ft-lb
D. $\frac{1}{4}$ ft-lb
E. 4 ft-lb

$$W = \int_a^b kx\,dx$$

$$36\text{ ft}\cdot\text{lb} = \int_0^{3\text{ft}} kx\,dx$$

$$= \left[\frac{kx^2}{2}\right]_0^3 = \frac{k}{2}\cdot 9$$

3 inches $= \frac{1}{4}$ ft

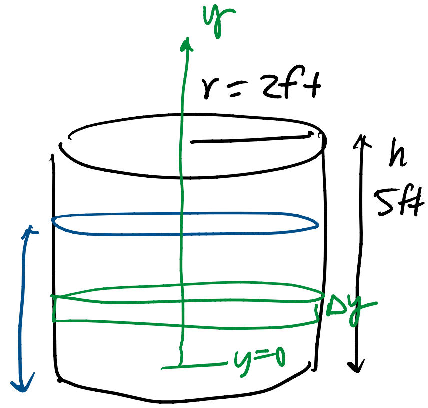
$$k = \frac{2\cdot 36}{9} = 2\cdot 4 = 8$$

$$W = \int_0^{\frac{1}{4}} 8x\,dx = \left[\frac{8x^2}{2}\right]_0^{\frac{1}{4}} = 4\left(\frac{1}{4}\right)^2 = \boxed{\frac{1}{4}\text{ ft lb}}$$

Pumping Problems:

$$W = F\cdot d = \underbrace{m\cdot g}_{\text{Force}}\cdot d$$

Fall 2018 #8



8. A cylindrical tank containing a liquid of density 10 lb/ft^3 has a radius of 2 ft. Its side is 5 ft. high and the depth of the liquid is 4 ft. How much work is required (in ft.-lbs.) to pump all the liquid out over the top of the tank?

A. 240π

B. $\boxed{480\pi}$

C. $1,000\pi$

D. 700π

E. 640π

$\rho = 10 \frac{lb}{ft^3}$ $\quad \frac{\text{force}}{\text{volume}}$

$W \sim \rho \cdot V \cdot d$

$$W = \int_0^4 \rho (\pi r^2 \cdot dy)(5-y)$$

$$= \pi \cdot 2^2 \cdot 10 \int_0^4 (5-y)dy = 40\pi \left[5y - \frac{y^2}{2}\right]_0^4$$

$$= 40\pi \left[20 - \frac{4^2}{2}\right] = 40\pi \cdot 12 = \boxed{480\pi}$$

In SI units: $\rho = \frac{kg}{m^3}$ $\quad \frac{\text{mass}}{\text{volume}}$

→ use gravity g

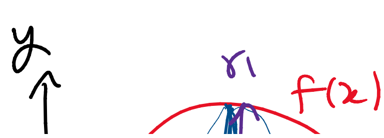
In imperical units $\rho = \frac{lb}{ft^3}$ $\quad \frac{\text{force}}{\text{volume}}$

→ don't use gravity

Another practice: Spring 2020 #12

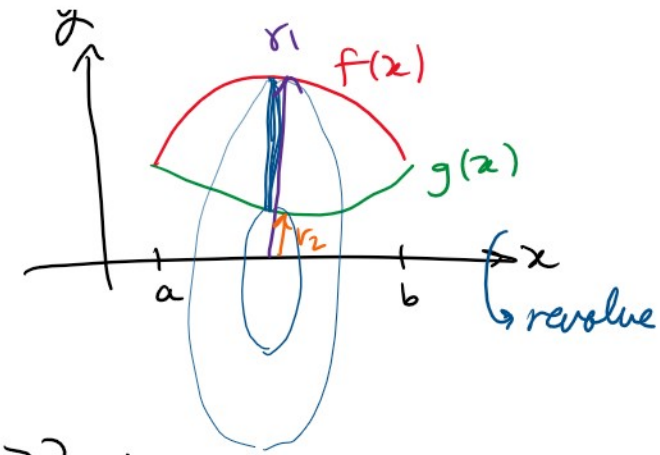
III. Washer vs. Shell vs. General Slicing

Integrate wrt x



Integral ...

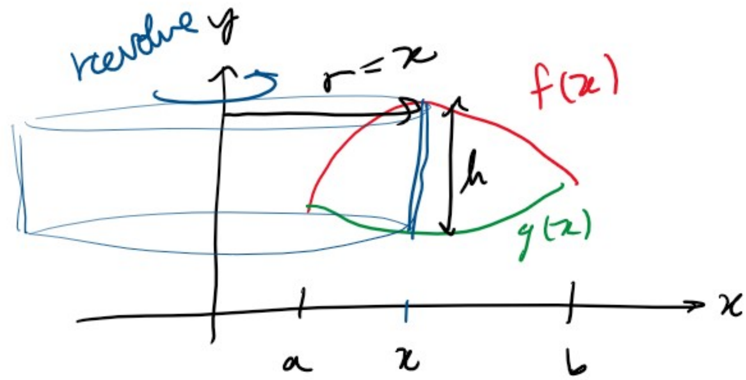
Washer:



$$V=\int_a^b \underbrace{\pi r_1^2}_{\text{outer radius}} - \underbrace{\pi r_2^2}_{\text{inner radius}}\, dx$$

$$= \int_a^b \pi\left(f(x)\right)^2 - \pi\left[g(x)\right]^2\, dx$$

Shell:



$$V=\int_a^b 2\pi r \cdot h\, dx$$

$$= \int_a^b \underbrace{2\pi x}_{\text{radius}} \cdot \underbrace{\left[f(x)-g(x)\right]}_{\text{height}} dx$$

Spring 2020 #8

8. Choose the right formula to compute the volume of the solid obtained from rotating the region bounded by $y = x^2, x = 2$ and $y = 0$ about $x = 3$ using

(i) Washer method,

(ii) Shell method.

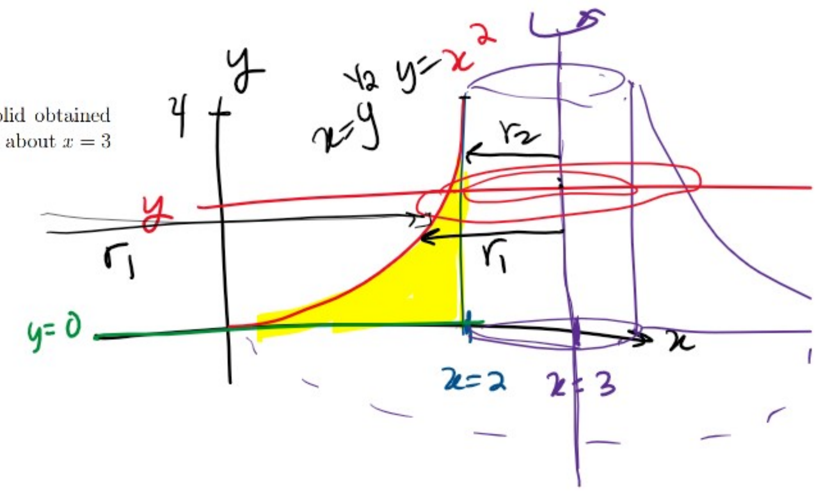
A. (i) $\int_0^4 \pi(\sqrt{y})^2 dy$, (ii) $\int_0^2 2\pi x \cdot x^2 dx$

B. (i) $\int_0^2 2\pi x \cdot x^2 dx$, (ii) $\int_0^4 \pi\left\{(\sqrt{y})^2 - 2^2\right\} dy$

C. (i) $\int_0^4 \pi\left\{(3-\sqrt{y})^2 - 1^2\right\} dy$, (ii) $\int_0^2 2\pi(3-x)x^2 dx$

D. (i) $\int_0^4 \pi\left\{(3+\sqrt{y})^2 - 1^2\right\} dy$, (ii) $\int_0^2 2\pi(x-3)x^2 dx$

E. (i) $\int_0^3 2\pi(3-x)x^2 dx$, (ii) $\int_0^4 \pi(3-\sqrt{y})^2 dy$



Washer is horizontal $\longrightarrow$ integrate wrt y

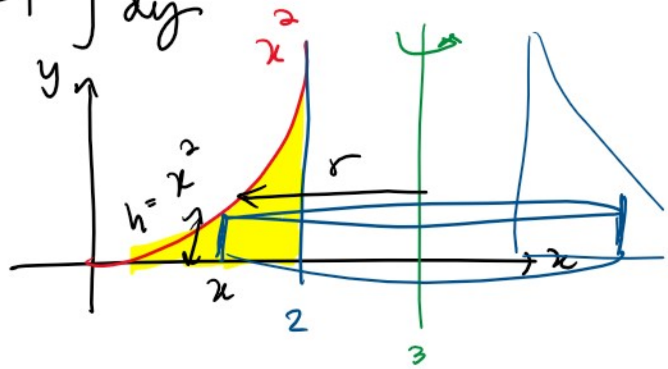
$V = \int^4 \pi r_1^2 - \pi r_2^2\, dy$ $\qquad r_1 = 3 - y^{1/2}$

Washer is horizontal

$$V = \int_0^4 \pi r_1^2 - \pi r_2^2 \, dy \qquad r_1 = 3 - y^{1/2}, \quad r_2 = 3 - 2 = 1$$

$$= \int_0^4 \pi (3 - y^{1/2})^2 - \pi (1)^2 \, dy$$

$$= \int_0^4 \pi \left[(3 - y^{1/2})^2 - 1 \right] dy$$



Shells: integrate wrt x

$$V = \int_0^2 2\pi r \cdot h \, dx$$

$$= \int_0^2 2\pi (3-x) \cdot x^2 \, dx$$

$r = 3 - x$

$h = x^2$

General Slicing:

Spring 2020 #9

9. The base of a solid is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$.

Find the volume of the solid if the cross-sections perpendicular to the base and parallel to the y-axis are equilateral triangles with one side lying along the base.

A. $\frac{1}{3}\sqrt{3}$

B. $\frac{16}{15}\sqrt{3}$

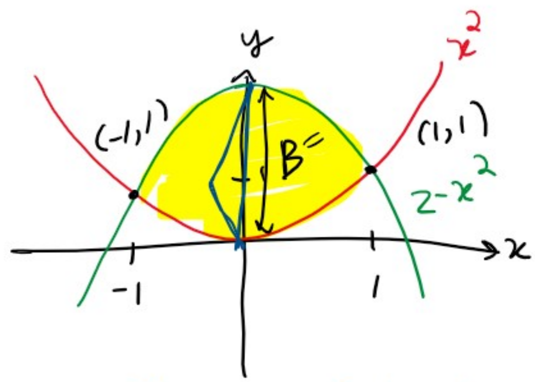
C. $\frac{8}{15}\sqrt{3}$

D. $\frac{3}{5}\sqrt{3}$

E. $\frac{6}{5}\sqrt{3}$

HINT: The area of an equilateral triangle with base length B is give by the formula $\frac{\sqrt{3}}{4}B^2$.

WARNING: The region above lies not only over the 1st quadrant but also over the 2nd quadrant.



equilateral

$$\text{area} = \frac{\sqrt{3}}{4} B^2$$

$$\int^1$$

$A(x) = \sqrt{3} \; B^2$

$$V = \int_{-1}^{1} A(x)\,dx$$

$$A(x) = \frac{\sqrt{3}}{4} B^2$$

$$B = 2 - x^2 - x^2 = 2 - 2x^2$$

$$V = \int_{-1}^{1} \frac{\sqrt{3}}{4}\left[2 - 2x^2\right]^2 dx$$

Spring 2020 #5

5. Find the angle θ between the following two tangent lines: one is to the curve $y = 3x^3$ at point $(1, 3)$ and the other to the curve $y = 3x^{\frac{1}{3}}$ at point $(1, 3)$.

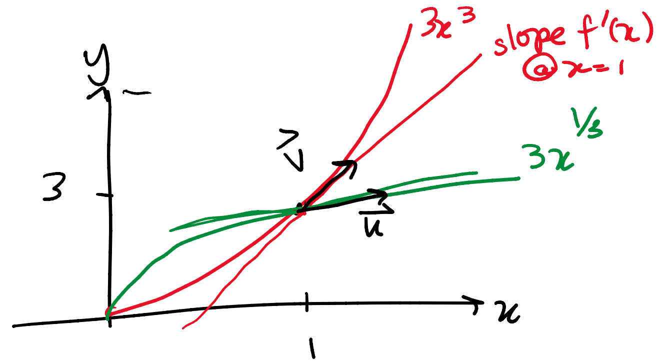
A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\arccos\left(\frac{3}{\sqrt{41}}\right)$

D. $\arccos\left(\frac{5}{\sqrt{35}}\right)$

E. $\arccos\left(\frac{5}{\sqrt{41}}\right)$



$$\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}| \cos(\theta)$$

① slope $= 3 \cdot 3x^2 \Big|_{x=1}$

$= 9 = \frac{\text{rise}}{\text{run}}$

$\vec{u} = \langle \text{run}, \text{rise} \rangle$

$= \langle 1, 9 \rangle$

② slope $= 3 \cdot \frac{1}{3} x^{-2/3} \Big|_{x=1}$

$= 1$

$\vec{v} = \langle 1, 1 \rangle$

$u \cdot v = 1 \cdot 1 + 1 \cdot 9$