Exam 1 Review - MA 166 LEC 286\

EXAM 1 - Today @ 6:30pm - 7:30pm in ELLT

No Lecture on Friday Feb 11

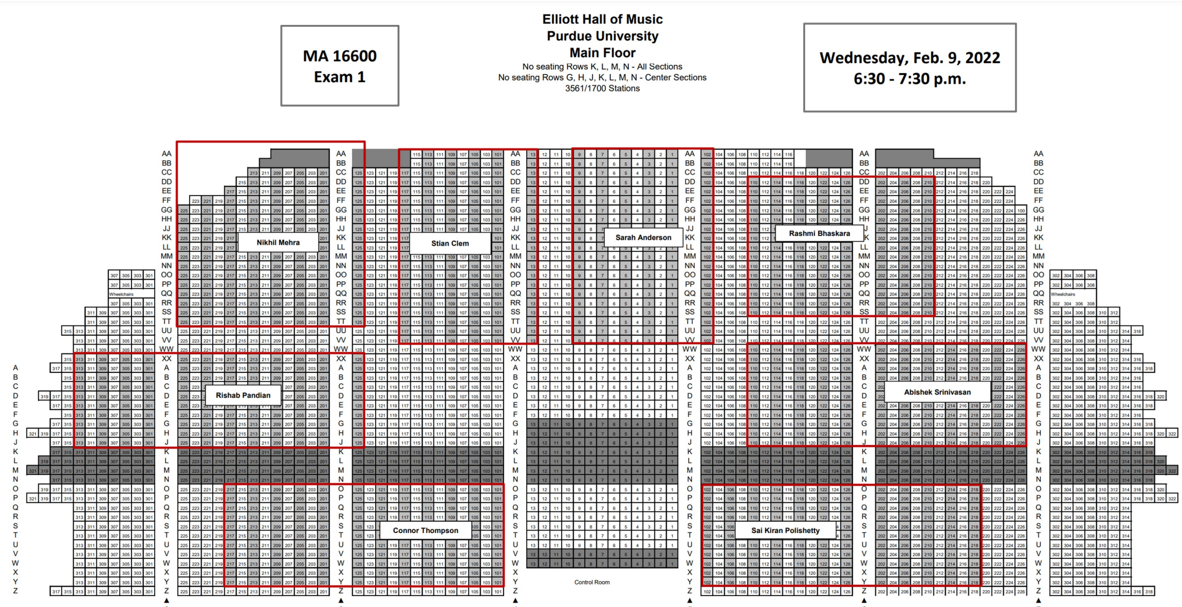
Instructions:

- please be in your seat by 6:15pm (so we can start right at 6:30pm)
- Know your recitation section #

1:30 pm Lecture		
TA	Time & Location	Recitation Number
Connor Thompson	10:30 am in PHYS 110	0208
Connor Thompson	11:30 am in PHYS 110	0215
Connor Thompson	12:30 pm in PHYS 110	0222
Nikhil Mehra	4:30 pm in PHYS 202	0229
Nikhil Mehra	3:30 pm in PHYS 202	0236
Nikhil Mehra	2:30 pm in PHYS 202	0243
Sarah Anderson	10:30 am in PHYS 202	0250
Sarah Anderson	9:30 am in PHYS 202	0257
Sarah Anderson	11:30 am in PHYS 202	0264
Rishab Pandian	2:30 pm in PHYS 110	0271
Rishab Pandian	1:30pm in PHYS 110	0278
Rishab Pandian	3:30 pm in PHYS 110	0285

2:30 pm Lecture		
TA	Time & Location	Recitation Number
Saikiran Polishetty	9:30 am in HAMP 2101	0287
Saikiran Polishetty	7:30 am in HAMP 2101	0288
Saikiran Polishetty	8:30 am in HAMP 2102	0289
Stian Clem	8:30 am in PHYS 333	0290
Stian Clem	7:30 am in PHYS 333	0291
Stian Clem	9:30 am in PHYS 333	0292
Abishek Srinivasan	3:30 pm in PHYS 333	0293
Abishek Srinivasan	4:30 pm in PHYS 333	0294
Abishek Srinivasan	2:30 pm in PHYS 333	0295
Rashmi Bhaskara	9:30 am in PHYS 110	0296
Rashmi Bhaskara	7:30 am in PHYS 110	0297
Rashmi Bhaskara	8:30 am in PHYS 110	0298

- sit in your designated area:



Bring:
- #2 pencil
- PUID
- Recitation section #
- TAs name

★ EXAM 1 Review: (Based on Hot Seat Poll)

1. Arc Length & Surface Area

* EXAM I REVIEW (Based ...)

1. Arc Length & Surface Area
2. Physical Applications
3. Shells vs. Washer vs. General Slicing Method
4. Lines, Planes, Spheres, & Vectors

I. Arc length & Surface Area :

Arc length L of $f(x)$ on $[a,b]$

$$L = \int_a^b \sqrt{1 + \{f'(x)\}^2}\, dx$$

Surface area S of the solid of revolution
$f(x)$ on $[a,b]$ revolved around x-axis

$$S = \int_a^b 2\pi f(x) \sqrt{1 + \{f'(x)\}^2}\, dx$$

Ex: $f(x) = \frac{x^4}{4} + \frac{1}{8x^2}$ on $[1,2]$

$$f'(x) = \frac{4x^3}{4} + \frac{1}{8}(-2)x^{-3} = x^3 - \frac{1}{4x^3}$$

$$L = \int_1^2 \sqrt{1 + \left\{x^3 - \frac{1}{4x^3}\right\}^2}\, dx$$

$$= \int_1^2 \sqrt{1 + x^6 - \frac{2\cdot\cancel{x^3}}{4\cancel{x^3}} + \frac{1}{16x^6}}\, dx$$

$$= \int_1^2 \sqrt{x^6 + \frac{1}{2} + \frac{1}{16x^6}}\, dx$$

$$= \int_1^2 \sqrt{\left(x^3 + \frac{1}{4x^3}\right)^2}\, dx$$

$$= \int_1^2 x^3 + \frac{1}{4x^3}\, dx = \left[\frac{x^4}{4} + \frac{1}{4}\frac{x^{-2}}{-2}\right]_1^2$$

$$= \int_1^2 x^3 + \frac{1}{4x^3}\, dx = \left[\frac{x^4}{4} + \frac{1}{4}\,\frac{x}{-2}\right]_1$$

$$= \left[\frac{x^4}{4} - \frac{1}{8x^2}\right]_1^2 = \cdots = \boxed{\frac{123}{32}}$$

Spring 2020 #11

11. Find the area of the northern cap of the sphere of radius 1 when the cap has height 1/2.

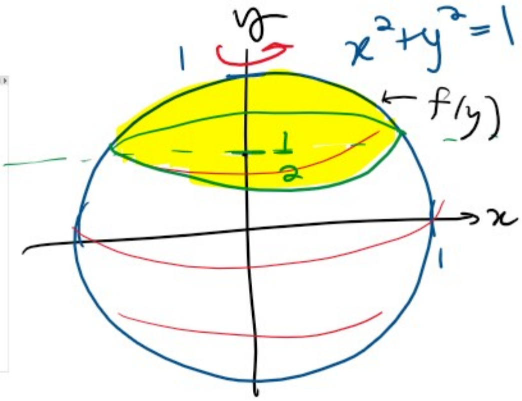
A. π

B. π^2

C. $\frac{4\pi}{3}$

D. $\frac{2\pi}{3}$

E. $\frac{\pi}{4}$



integrate wrt y

$$S = \int_{\frac{1}{2}}^{1} 2\pi f(y) \sqrt{1 + \{f'(y)\}^2}\, dy$$

$$f(y) = \sqrt{1-y^2} \qquad f'(y) = \frac{1}{2}(1-y^2)^{-1/2}(-2y)$$

$$= \frac{-y}{\sqrt{1-y^2}}$$

$$S = \int_{\frac{1}{2}}^{1} 2\pi \sqrt{1-y^2} \cdot \sqrt{1 + \left\{\frac{-y}{\sqrt{1-y^2}}\right\}^2}\, dy$$

$$= \int_{\frac{1}{2}}^{1} 2\pi \sqrt{1-y^2} \sqrt{\frac{1-\cancel{y^2}}{1-y^2} + \frac{\cancel{y^2}}{1-y^2}}\, dy$$

$$= \int_{\frac{1}{2}}^{1} 2\pi \cancel{\sqrt{1-y^2}} \cdot \frac{1}{\cancel{\sqrt{1-y^2}}}\, dy = \int_{\frac{1}{2}}^{1} 2\pi\, dy$$

$$= \left[2\pi y\right]_{\frac{1}{2}}^{1} = 2\pi - \pi = \boxed{\pi} \quad \boxed{A}$$

II. Physical Applications:

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Spring — Hooke's Law $F = kx$ x is a displacement from the natural length

Then $W = \int_a^b kx\,dx$

Spring 2018 #7

7. If the work required to stretch a spring 3 ft beyond natural length is 36 ft-lb, how much work is needed to stretch it 3 inches beyond natural length ?

A. $\frac{3}{8}$ ft-lb

B. 54 ft-lb

C. 36 ft-lb

D. $\frac{1}{4}$ ft-lb

E. 4 ft-lb

$$W = \int_a^b kx\,dx$$

$$36 \text{ ft-lb} = \int_0^{3ft} k\,x\,dx = \left[\frac{kx^2}{2}\right]_0^3$$

$$36 = \frac{k \cdot 9}{2} \rightarrow k = \frac{2 \cdot 36}{9} = \boxed{8 = k}$$

$$W = \int_0^{\frac{1}{4}} 8x\,dx = \left[\frac{8x^2}{2}\right]_0^{\frac{1}{4}} = \cancel{4} \cdot \left(\frac{1}{4}\right)^{\cancel{2}} = \boxed{\frac{1}{4} \text{ ft-lb}}$$

- Pumping Problems:

$$W \sim F \cdot d = \rho \cdot V \cdot g \cdot d$$

Fall 2018 #8

8. A cylindrical tank containing a liquid of density 10 lb/ft^3 has a radius of 2 ft. Its side is 5 ft. high and the depth of the liquid is 4 ft. How much work is required (in ft.-lbs.) to pump all the liquid out over the top of the tank?

A. 240π

B. 480π

C. $1,000\pi$

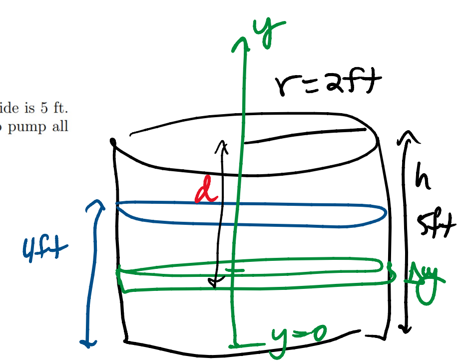
D. 700π

E. 640π

$\rho = 10 \text{ lb/ft}^3$

units $\frac{\text{Force}}{\text{Volume}}$

$F = \rho V$



$$W \sim F \cdot d = \rho \cdot \underbrace{(\pi r^2 \cdot \Delta y)}_{\text{volume}}(5-y)$$

$$\int_0^4 \rho\pi r^2 (5-y)\,dy = 10 \cdot \pi (2)^2 \int^4 (5-y)\,dy$$

$$W = \int_0^4 \rho \pi r^2 (5-y)\,dy = 10\cdot\pi(2)^2 \int_0^4 (5-y)\,dy$$

$$= 40\pi \left[5y - \frac{y^2}{2}\right]_0^4 = 40\pi\left[20 - \frac{4^2}{2}\right]$$

$$= 40\pi \cdot 12 = \boxed{480\pi}$$

NOTE:

If in impirical units $\rho = \frac{lbs}{ft^3}$ – $\frac{\text{Force}}{\text{Volume}}$

Don't use g

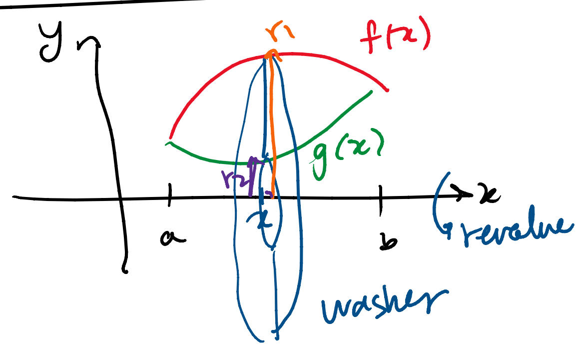
If in SI units $\rho = \frac{kg}{m^3}$ – $\frac{\text{mass}}{\text{volume}}$

$F = g \cdot \rho \cdot V \rightarrow$ use g

Similar – Spring 2020 #12

III. Washer vs. Shells vs. Slicing Method

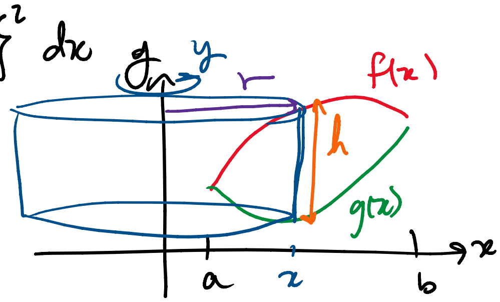
integrate wrt x



Washer Method

$$V = \int_a^b \left(\pi \underbrace{r_1^2}_{\text{out radius}} - \pi \underbrace{r_2^2}_{\text{inner radius}}\right) dx$$

$$= \int_a^b \pi\{f(x)\}^2 - \pi\{g(x)\}^2\,dx$$



Shell Method

$$V = \int_a^b 2\pi \underbrace{r}_{\text{radius}} \cdot \underbrace{h}_{\text{height}}\,dx$$

$$= \int_a^b 2\pi x \cdot [f(x) - g(x)]\,dx$$

Spring 2020 #8

8. Choose the right formula to compute the volume of the solid obtained from rotating the region bounded by $y = x^2, x = 2$ and $y = 0$ about $x = 3$ using

(i) Washer method,

(ii) Shell method.

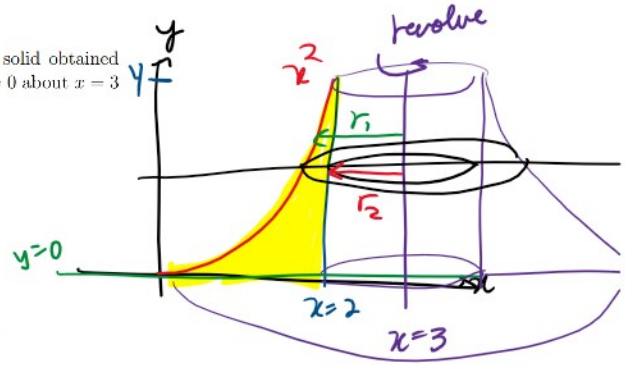
A. (i) $\int_0^4 \pi(\sqrt{y})^2 dy$, (ii) $\int_0^2 2\pi x \cdot x^2 dx$

B. (i) $\int_0^2 2\pi x \cdot x^2 dx$, (ii) $\int_0^4 \pi \{(\sqrt{y})^2 - 2^2\} dy$

C. (i) $\int_0^4 \pi \{(3 - \sqrt{y})^2 - 1^2\} dy$, (ii) $\int_0^2 2\pi(3 - x)x^2 dx$

D. (i) $\int_0^4 \pi \{(3 + \sqrt{y})^2 - 1^2\} dy$, (ii) $\int_0^2 2\pi(x - 3)x^2 dx$

E. (i) $\int_0^3 2\pi(3 - x)x^2 dx$, (ii) $\int_0^4 \pi(3 - \sqrt{y})^2 dy$



$y = x^2 \rightarrow x = y^{1/2}$

Washer: horizontal slice $\longrightarrow$ washer
integrate wrt y

$r_1 = 3 - f(y)$

$r_1 = 3 - y^{1/2}$

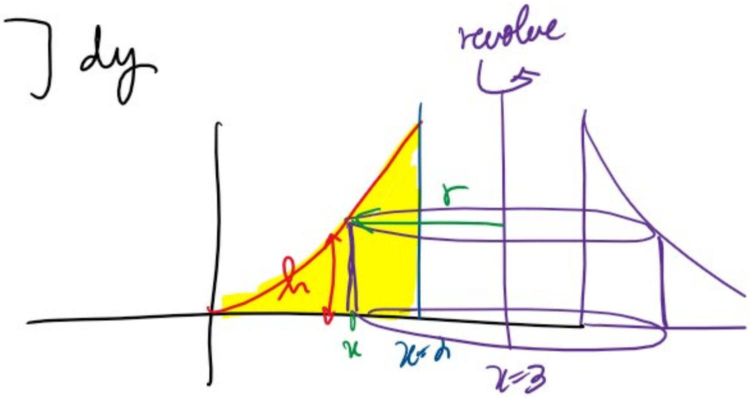
$r_2 = 3 - 2 = 1$

$$V = \int_0^4 \pi r_1^2 - \pi r_2^2 \, dy$$

$$V = \int_0^4 \pi \left[3 - y^{1/2}\right]^2 - \pi (1)^2 dy$$

$$= \int_0^4 \pi \left[(3 - y^{1/2})^2 - 1 \right] dy$$

Shell:
cylinder - integrate wrt x



$$V = \int_0^2 2\pi r \cdot h \, dx$$

$r = 3 - x$

$h = x^2$

$$= \int_0^2 2\pi (3 - x) \cdot x^2 dx$$

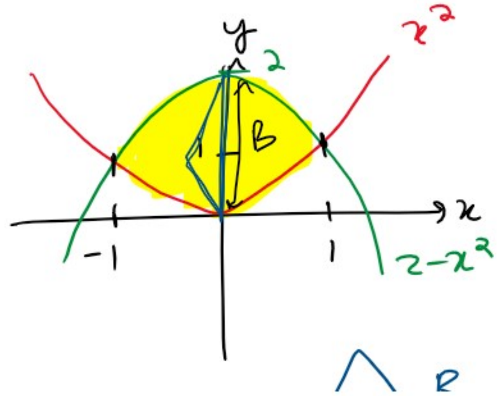
Spring 2020 #9

9. The base of a solid is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$.

Find the volume of the solid if the cross-sections perpendicular to the base and parallel to the y-axis are equilateral triangles with one side lying along the base.

A. $\frac{1}{3}\sqrt{3}$

16



$B = 2 - x^2 - x^2$

and parallel to the y-axis are equilateral triangles with one side lying along the base.

A. $\frac{1}{3}\sqrt{3}$

B. $\frac{16}{15}\sqrt{3}$

C. $\frac{8}{15}\sqrt{3}$

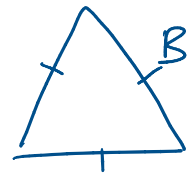
D. $\frac{3}{5}\sqrt{3}$

E. $\frac{6}{5}\sqrt{3}$

HINT: The area of an equilateral triangle with base length B is give by the formula $\frac{\sqrt{3}}{4}B^2$.

WARNING: The region above lies not only over the 1st quadrant but also over the 2nd quadrant.

cross-section



$B = 2 - x^2 - x^2$

$B = 2 - 2x^2$

area $= \frac{\sqrt{3}}{4} B^2 = A(x)$

Slicing Method $\quad V = \int_{-1}^{1} A(x)\,dx$

$$A(x) = \frac{\sqrt{3}}{4} B^2 = \frac{\sqrt{3}}{4}\left[2 - 2x^2\right]^2$$

$$V = \int_{-1}^{1} \frac{\sqrt{3}}{4}\left[2 - 2x^2\right]^2 dx \;\ldots\; = \frac{16\sqrt{3}}{15}$$