Exam 3 Review - Monday - LEC 200

In Hot Seat Thoughts, you can request topics/problems to review on Wednesday

Announcements:

New calendar dates:

HW 33 due Mon 4/25

HW 34 due Wed 4/27

Office Hours tomorrow Tues @ 1-2pm

Zoom link on LEC Brightspace

★Warm Up:

Is the following statement true or false?

If a sequence a_n is such that $-n-1 \le \ln|a_n| \le -n$ then $\sum_{n=1}^{\infty} a_n$ converges

(From S19 - Exam 3 - #7 part III)

(Hint: use the Root Test)

(a) True (b) False

Root Test

$$\rho = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} |a_n|^{1/n}$$

$$= e^{\lim_{n\to\infty} \ln(|a_n|^{1/n})} = e^{\lim_{n\to\infty} \frac{1}{n}\ln|a_n|} = e^{-1}$$

$$-1-\frac{1}{n} \le \frac{-n-1}{n} \le \frac{\ln|a_n|}{n} \le \frac{-n}{n} = -1$$

$$\frac{\ln|a_n|}{n} \xrightarrow{n\to\infty} -1$$

$$\rho = \frac{1}{e} < 1$$

$\to \sum a_n$ converges

NOTE: We did not cover multiplication of power series. In past exams, skip

$\int e^{-2x}\sin(3x)$

F15 #11

F11 #11

SKIP $\begin{cases} e^{-2x}\sin(3x) \\ \sin(x)\cos(x) \\ \tan(x) = \dfrac{\sin(x)}{\cos(x)} \end{cases}$

F15 #11

F16 #11

F16 #12

Computing the value of series

S13 - Exam 3 - #1 :

Compute $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n-1} - 3^{n+1}}{4^n}$

separate : $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n-1}}{4^n} - \sum_{n=1}^{\infty} \frac{3^{n+1}}{4^n}$

Geometric series $\displaystyle\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$ if $|r|<1$

Convert $k = n-1$

$k+1 = n$

$$= \sum_{k=0}^{\infty} \frac{2^k}{4^{k+1}} - \sum_{k=0}^{\infty} \frac{3^{k+2}}{4^{k+1}}$$

$$= \sum_{k=0}^{\infty} \underbrace{\frac{1}{4}}_{a} \underbrace{\left(\frac{2}{4}\right)}_{\substack{r \\ <1}}^k - \sum_{k=0}^{\infty} \underbrace{\frac{3^2}{4}}_{a} \underbrace{\left(\frac{3}{4}\right)}_{r<1}^k$$

$$= \frac{\left(\frac{1}{4}\right)}{1-\left(\frac{1}{2}\right)} - \left(\frac{\frac{9}{4}}{1-\frac{3}{4}}\right) = \frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} - \frac{\left(\frac{9}{4}\right)}{\left(\frac{1}{4}\right)} = \frac{1}{2} - 9$$

$$= \boxed{\frac{-17}{2}}$$

$\boxed{D}$

Integrals of Power Series

518 - Exam 3 - #10

Find The power series $f(x)$ for $\int \frac{x^2}{1+4x^2}\,dx$ centered at 0, and find the Radius of Conv.

$$\int \frac{x^2}{1+4x^2}\,dx = \int x^2\left(\frac{1}{1+4x^2}\right)dx$$

let $r = -4x^2$

Common power series $\quad \frac{1}{1-r} = \sum_{k=0}^{\infty} r^k$

$$\frac{1}{1+4x^2} = \frac{1}{1-(-4x^2)} = \sum_{k=0}^{\infty}(-4x^2)^k$$

$$= \sum_{k=0}^{\infty}(-1)^k 4^k x^{2k}$$

$$= \int x^2\left(\sum_{k=0}^{\infty}(-1)^k 4^k x^{2k}\right)dx = \int \sum_{k=0}^{\infty}(-1)^k 4^k x^{2k+2}\,dx$$

$$= \sum_{k=0}^{\infty}(-1)^k 4^k \int x^{2k+2}\,dx$$

$$f(x) = \left(\sum_{k=0}^{\infty}(-1)^k 4^k \frac{x^{2k+3}}{2k+3}\right) + C$$

Radius of convergence:

If $f(x)$ has radius of conv. R

then $\int f(x)\,dx$ also has rad. of. conv. R

Geometric series

$$|r| < 1$$

$$|-4x^2| < 1$$

$$4|x|^2 < 1$$

$$\sqrt{|x|^2} \leq \sqrt{\frac{1}{4}}$$

$$|x| < \frac{1}{2}$$

$$-\frac{1}{2} < x < \frac{1}{2}$$

$$R = \frac{\left(\frac{1}{2} - \left(-\frac{1}{2}\right)\right)}{2} = \frac{1}{2}$$

Convert functions $\longrightarrow$ Power Series

$$f(x) \longrightarrow \sum_{k=0}^{\infty} c_k (x-a)^k$$

Common Maclaurin series:

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

Geometric series

$$e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k$$

B/c $\frac{d^k}{dx^k}(e^x) = e^x$

$$\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cos(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

Will be given on the exam in the statement of problem

Ex: $\ln|1-x| = -\int \frac{dx}{1-x}$

$u = 1-x$

$du = -dx$

$$\sum^{\infty} \frac{x^{k+1}}{} + C$$

$$= -\int \sum_{k=0}^{\infty} x^k \, dx = -\sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1} + C$$

@ $x=0$ $$\ln|\cancel{1-0}| = -\cancel{\sum_{k=0}^{\infty} \frac{0^{k+1}}{k+1}} + C \rightarrow \boxed{C=0}$$

$$\boxed{\ln|1-x| = -\sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1}}$$

Ex: $\dfrac{e^x - 1}{x}$

Power Series $\longrightarrow$ Function

look for common Maclaurin series

Ex: $$\sum_{k=0}^{\infty} 2^k x^{2k+1} = \sum_{k=0}^{\infty} (2x^2)^k (x)$$

$$= (x)\Big(\underbrace{\sum_{k=0}^{\infty} (2x^2)^k}_{\substack{\text{Geometric} \\ r = 2x^2}}\Big)$$

$$= x\left(\frac{1}{1-2x^2}\right) = \frac{x}{1-2x^2}$$

If-Then Type Problems: Which of the following are true?

∞ ✓ ... then $\sum^{\infty} (-1)^n a_n$ ✗

① If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges ✗

To be an alternating series need $a_n \geq 0$

Counter example:

$a_n = \frac{(-1)^{n+1}}{n}$ A.S. converges

$$(-1)^n a_n = \frac{(-1)^n(-1)^{n+1}}{n} = \frac{(-1)^{2n+1}}{n} = \frac{-1}{n}$$

$$\sum (-1)^n a_n = -\left(\sum \frac{1}{n}\right) \longrightarrow \text{diverges}$$

FALSE

HOTSEAT:

If $\sum_{n=1}^{\infty} |a_n|$ converges $\Rightarrow \sum_{n=1}^{\infty} (-1)^n a_n$ converges

TRUE:

Comparison Test

$(-1)^n a_n \leq |a_n|$

↳ converges

By Com. Test $\sum_{n=1}^{\infty} (-1)^n a_n$ converges

③ $\lim_{n\to\infty} n^2 |a_n| = 1 \Rightarrow \sum_{n=1}^{\infty} (-1)^n a_n$ absolutely converges

Want to show $\sum_{n=1}^{\infty} |(-1)^n a_n| = \sum_{n=1}^{\infty} |a_n|$ converges

L.C.T. $b_n = \frac{1}{n^2}$

$\lim \frac{|a_n|}{\ } = 1$ $\qquad b_n = \frac{1}{n^2}$ p-series $p=2$

L.C.T. $b_n =$ ...

$$L = \lim_{n\to\infty} \frac{|a_n|}{\frac{1}{n^2}} = 1$$

$b_n = \frac{1}{n^2}$ p-series $p=2$ converge

$\Rightarrow \sum |a_n|$ converges

(4) The series $\sum_{n=1}^{\infty} (-1)^n \sqrt{\frac{3n+2}{4n-1}}$ converges.

Alternating series $\longrightarrow$ A.S.T.

(a) nonincreasing

(b) $\lim_{n\to\infty} \sqrt{\frac{3n+2}{4n-1}} = \sqrt{\frac{3}{4}} \neq 0$

FALSE