Final Exam Review - Friday - LEC 200

Announcements:

Final Exam Mon May 2nd @ 10:30am – 12:30pm in ELLT

Office Hour @ 4-5pm in MATH 817

Warm Up: Make the appropriate u-substitution to write the integral in terms of u:

$$\int \sin^{2/3}(x)\cos^3(x)\,dx \qquad (F19\#4)$$

(a) $\int u^{8/3}\,du$

(b) $\boxed{\int \left(u^{2/3} - u^{8/3}\right) du}$

(c) $\int \left(u^{2/3} + u^{8/3}\right) du$

$\cos^3(x) \longrightarrow$ $du = \cos(x)dx$

$u = \sin(x)$

$$\int \underbrace{\sin^{2/3}(x)}_{u^{2/3}}\ \underbrace{\cos^2(x)}_{1-\sin^2(x)}\ \underbrace{\cos(x)dx}_{du}$$

$$= \int u^{2/3}(1-u^2)\,du = \int u^{2/3} - u^{8/3}$$

F19#4: $\int_0^{\frac{\pi}{2}} \ldots\, dx$

Series Questions:

F18#15: The quantity $(\cos 2x)\sum_{n=0}^{\infty}(\tan x)^{2n}$ $\rightarrow n=1$ for $0 \le x < \frac{\pi}{4}$ is equal to ?

Rewrite $\sum_{n=0}^{\infty}(\tan x)^{2n}$ $\rightarrow n=1$ $= \sum_{n=0}^{\infty}(\tan^2 x)^n$ $\rightarrow n=1$

Geometric Series $r = \tan^2 x$

Rewrite $\sum (\tan x)$ $\quad \sum$ $\quad r = \tan^2 x$

$n=0 \to n=1$ $\quad n=0 \to n=1$

$$= \left(\frac{1}{1-\tan^2(x)}\right) - 1$$

write in terms of $\cos(x)$

$\tan^2(x) = \sec^2(x) - 1$

$$= \left(\frac{1}{2-\sec^2(x)} \cdot \frac{\cos^2(x)}{\cos^2(x)}\right) - 1$$

$1 - (\sec^2(x) - 1)$

$= 1 - \sec^2(x) + 1$

$$= \left(\frac{\cos^2(x)}{2\cos^2(x) - 1}\right) - 1$$

Half angle formula

$\cos^2(x) = \dfrac{1+\cos(2x)}{2}$

$2\cos^2(x) = 1 + \cos(2x)$

$2\cos^2(x) - 1 = \cos(2x)$

$$= \left(\frac{\cos^2(x)}{\cos(2x)}\right) - 1$$

side note series starts at $n=1$ answer is $\sin^2(x)$

$$(\cos 2x) \sum_{n=0}^{\infty} (\tan x)^{2n} = \cancel{\cos(2x)}\left(\frac{\cos^2(x)}{\cancel{\cos(2x)}} - 1\right) = \cos^2(x) - 1$$

S19 #21: Use formulas and the Alternating Series Estimation Theorem to compute

$$\int_0^{0.1} \ln(1+x)\,dx \text{ with error} \leq 10^{-4}$$

Break into 2 Questions

Q1: Compute $\int_0^{0.1} \ln(1+x)\,dx$

$$\int_0^{0.1} \ln(1+x)\,dx = \int_0^{0.1} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n!}\,dx$$

$$= \sum \frac{(-1)^{n-1}}{} \int_0^{0.1} x^n\,dx = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!} \left[\frac{x^{n+1}}{n+1}\right]_0^{0.1}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!} \int_0^{0.1} x^n \, dx = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!} \left(\frac{x^{n+1}}{n+1} \right]_0^{0.1}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\underbrace{n!\,(n+1)}_{(n+1)!}} \left[\underbrace{(0.1)^{n+1}}_{\left(\frac{1}{10}\right)} - \cancel{0^{n+1}}^{\,0} \right]$$

$$= \boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n+1)!\, 10^{n+1}}}$$

HOTSEAT: Use the Alternating Series Estimation Thm to determine how many terms you need to compute $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n+1)!\, 10^{n+1}}$ with error $< 10^{-4}$

(a) 1

$\boxed{\text{(b) } 2}$

(c) 3

(d) 4

$$|R_k| \leq a_{k+1} = \frac{1}{(k+2)!\, 10^{k+2}} \leq \frac{1}{10^4}$$

$k+2 = 4 \qquad k=2$

$$|R_2| = \frac{1}{4!\, 10^4} < \frac{1}{10^4}$$

we need $\boxed{k=2}$ terms

check if $k=1$ $\quad |R_1| \leq \frac{1}{3!\, 10^3} = \frac{1}{6 \cdot 10^3} > \frac{1}{10^4}$

Now we can compute

$$\int_0^{0.1} \ln(1+x)\, dx \approx \sum^{2} \frac{(-1)^{n-1}}{(n+1)!\, 10^{n+1}}$$

Now we ...

$$\int_0^{0.1} \ln(1+x)\,dx \cong \sum_{n=1}^{\infty} \frac{(-1)}{(n+1)!\,10^{n+1}}$$

$$= \frac{(-1)^0}{2!\,10^2} + \frac{(-1)^1}{3!\,10^3} = \boxed{\frac{1}{200} - \frac{1}{6000}}$$

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index @ $k=0$ $\longrightarrow$ Geometric series

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$$

$\searrow$ Taylor series

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

<u>S19 #17:</u> Compute the limit

$$\lim_{n\to\infty} \left(\sqrt{\underbrace{n^4+n^3+n^2}_{A}} - \sqrt{\underbrace{n^4+n^3+2n^2+1}_{B}} \right)$$

$$\lim_{n\to\infty} \sqrt{A} - \sqrt{B} \left(\frac{\sqrt{A}+\sqrt{B}}{\sqrt{A}+\sqrt{B}} \right) = \lim_{n\to\infty} \frac{A-B}{\sqrt{A}+\sqrt{B}}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^4}+\cancel{n^3}+n^2-(\cancel{n^4}+\cancel{n^3}+2n^2+1)}{\sqrt{A}+\sqrt{B}}$$

$$= \lim_{n\to\infty} \frac{-n^2-1}{\sqrt{n^4+n^3+n^2}+\sqrt{n^4+n^3+2n^2+1}} \quad \frac{\left(\frac{1}{n^2}\right)}{\left(\frac{1}{n^2}\right)}$$

$$= \lim_{n\to\infty} \frac{-n^2-1}{\sqrt{n^4+n^3+n^2}+\sqrt{n^4+n^3+2n^2+1}} \quad \left(\frac{1}{n^2}\right)$$

$$= \lim_{n\to\infty} \frac{-1-\frac{1}{n^2}}{\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}+\frac{2}{n^2}+\frac{1}{n^4}}} = \frac{-1}{\sqrt{1}+\sqrt{1}} = \boxed{-\frac{1}{2}}$$

Polar Coordinates:

1. $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ $\begin{cases} r = \sqrt{x^2+y^2} \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) \end{cases}$

2. plotting polar functions $r = f(\theta)$

   plot r vs. θ $\longrightarrow$ plot points in the x-y plane

3. Slope of line tangent to $r = f(\theta)$

   $x = f(\theta)\cos\theta$
   $y = f(\theta)\sin\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

4. Area $r = f(\theta)$ over $[\alpha, \beta]$

$$A = \frac{1}{2}\int_\alpha^\beta [f(\theta)]^2\, d\theta$$

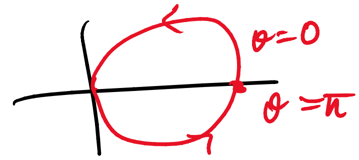
5. Arc Length $r = f(\theta)$ over $[\alpha, \beta]$

$$L = \int_\alpha^\beta \sqrt{f(\theta)^2 + \{f'(\theta)\}^2}\, d\theta$$

... $r = 6\cos\theta$

Find the arc length of a circle $r = 6\cos\theta$

$[0, \pi]$



HOTSEAT: (S20 # 11)

Find the slope of line tangent to $r = 8\sin\theta$ @ point $(4, \frac{\pi}{6})$

(a) $\sqrt{3}$

(b) $2\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$

$$\frac{dy}{dx} = \left.\frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}\right|_{\theta = \frac{\pi}{6}}$$

$f(\frac{\pi}{6}) = 4$

$f'(\frac{\pi}{6}) = 8\cos\theta\big|_{\frac{\pi}{6}} = 4(\frac{\sqrt{3}}{2}) = 4\sqrt{3}$

$\sin(\frac{\pi}{6}) = \frac{1}{2}$

$\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

$$\frac{dy}{dx} = \frac{(4\sqrt{3})(\frac{1}{2}) + (4)(\frac{\sqrt{3}}{2})}{(4\sqrt{3})(\frac{\sqrt{3}}{2}) - (4)(\frac{1}{2})} = \sqrt{3}$$