Announcements:

Final Exam - Mon May 2nd @ 10:30 am - 12:30pm in ELLT

Office Hours: Friday @ 11am - noon on zoom

@ 4 - 5pm in MATH 817

☆ Warm Up: Find the arc length of the circle $r = 6\cos\theta$ for $0 \le \theta \le \frac{\pi}{3}$

(a) 2π

(b) 6π

(c) 4π

(d) 3π

$$L = \int_0^{\frac{\pi}{3}} \sqrt{f(\theta)^2 + f'(\theta)^2}\, d\theta$$

Exam 3: Benchmark

→ Booklets returned in REC tomorrow

→ Answer Key tomorrow

Final Exam: 120 minutes

25 Qs × 8 pts = 200 pts

cumulative exam: lessons 1 - 34

Exam Breakdown:

7 Qs - Exam 1 (lessons 1 - 9) vectors Areas & Volms

6 Qs - Exam 2 (lessons 10 - 18) Integration

9 Qs - Exam 3 (lesson 19 - 31) series power series

3 Qs - polar coordinates

NOT cover:

- center of mass / centroids
- parametric equations
- complex numbers

Equation List $\longrightarrow$ cover sheet

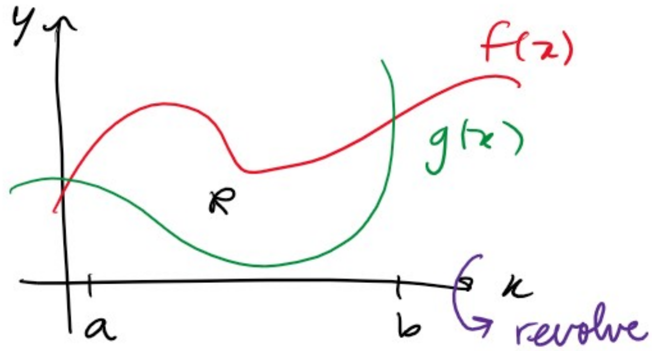
same assigned seats

LEC Brightspace $\rightarrow$ Gradebook

Find volumes of solids of revolution

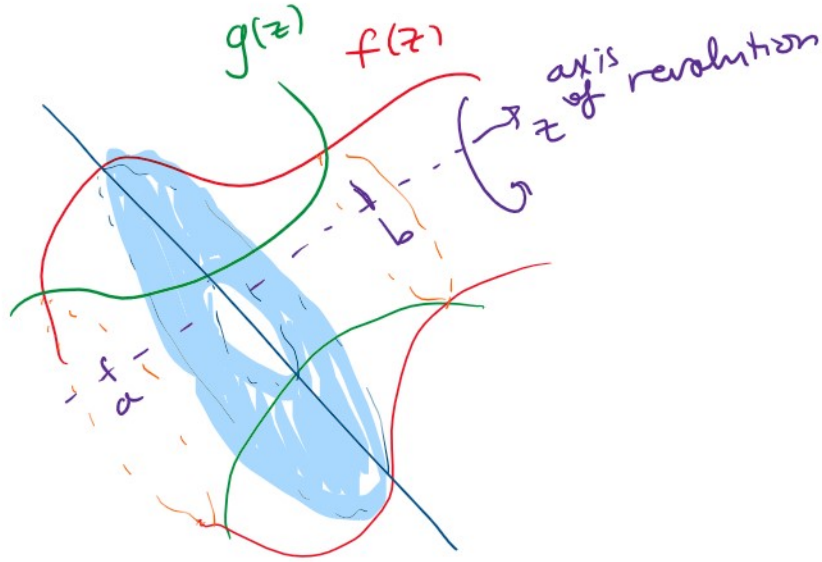
Washer Method:

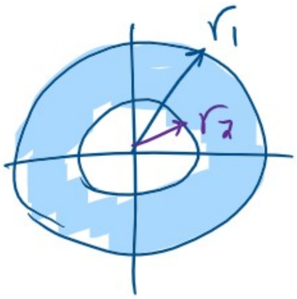
$$V = \int_a^b \pi \left[f(x)^2 - g(x)^2 \right] dx$$



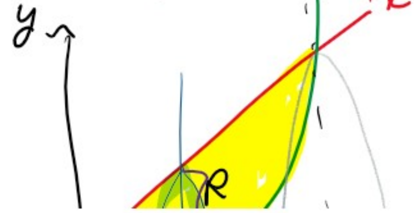
In general

$$V = \int_a^b \pi \left[r_1^2 - r_2^2 \right] dz$$

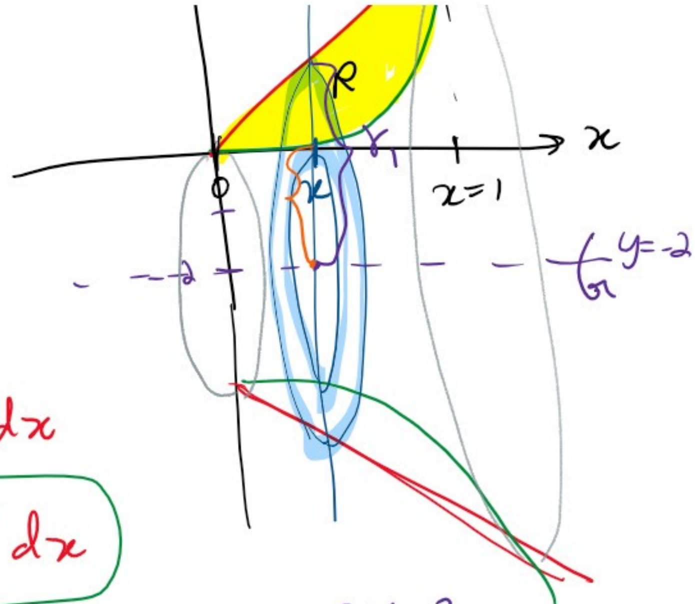




HOTSEAT: Let R be bounded by $y = x$, $y = x^2$, $x = 0$, $x = 1$.
Revolve R around



Revolve R around the line $y=-2$



Use Washer Method to set up the integral for the volume

(a) $V=\pi\int_0^1 (x-2)^2-(x^2-2)^2\,dx$

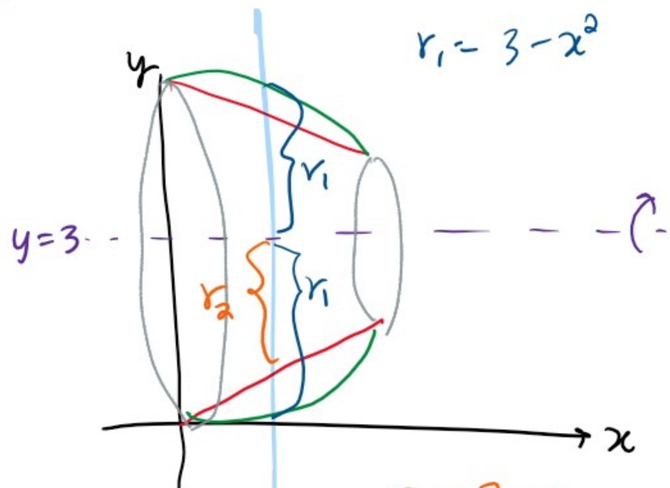
(b) $V=\pi\int_0^1 (x+2)^2-(x^2+2)^2\,dx$

(c) $V=\pi\int_0^1 x^2-x^4\,dx$

$r_1=x+2$

$r_2=x^2+2$

HOTSEAT: Same R, but now revolve around $y=+3$



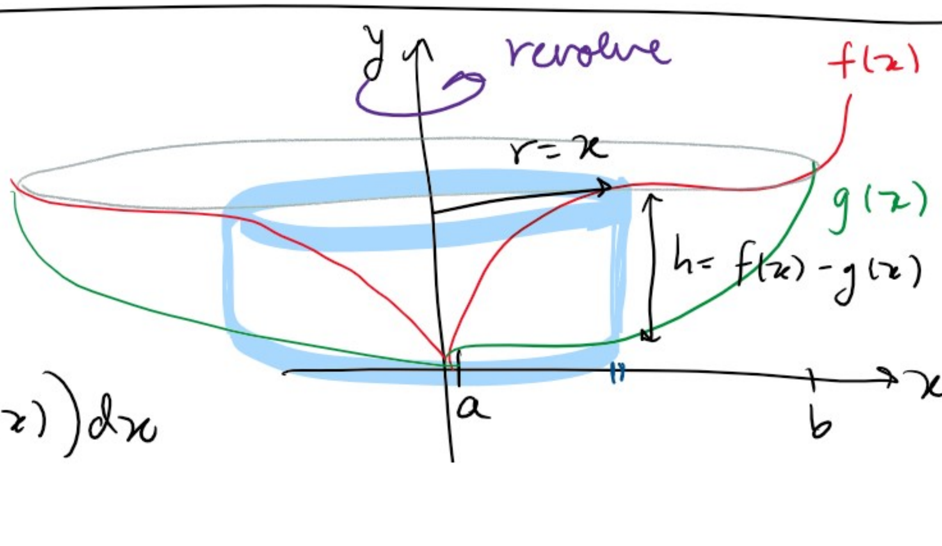
(a) $V=\pi\int_0^1 (3-x^2)^2-(3-x)^2\,dx$

(b) $V=\pi\int_0^1 (3+x^2)^2-(3+x)^2\,dx$

(c) $V=\pi\int_0^1 x^2-x^4\,dx$

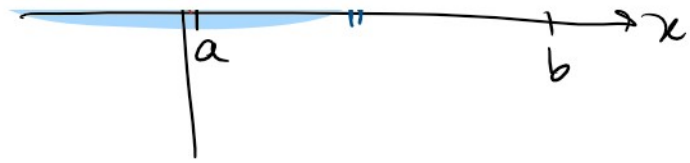
$r_2=3-x$

Shell Method:



$V=\int^b 2\pi x\,\big(f(x)-g(x)\big)\,dx$

$$V = \int_a^b 2\pi x\,\big(f(x) - g(x)\big)\,dx$$



HOTSEAT: R – $y = x$, $y = x^2$, $x = 0$, $x = 1$

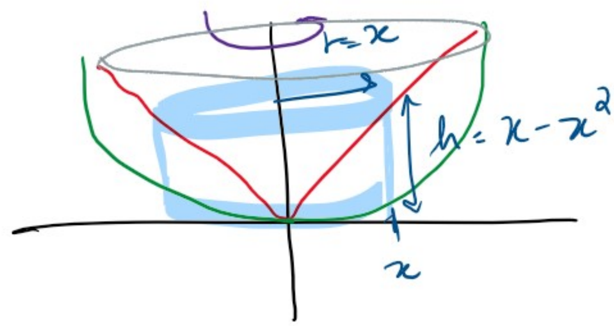
revolve around the y-axis

Use the Shell Method to set up the V integral

(a) $V = \int_0^1 2\pi x\,(x)\,dx$

(b) $V = \int_0^1 2\pi x\,(x^2 - x)\,dx$

(c) $V = \int_0^1 2\pi x\,(x - x^2)$

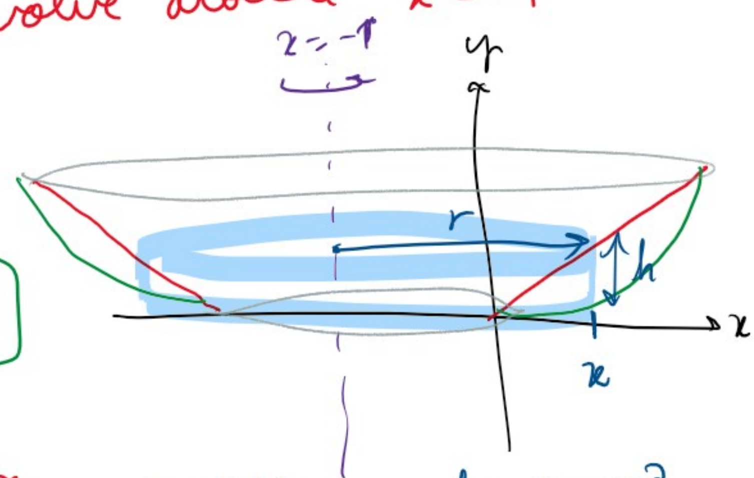


HOTSEAT: Same R, revolve around $x = -1$

(a) $V = \int_0^1 2\pi x\,(x - x^2)\,dx$

(b) $V = \int_0^1 2\pi (x+1)(x - x^2)\,dx$

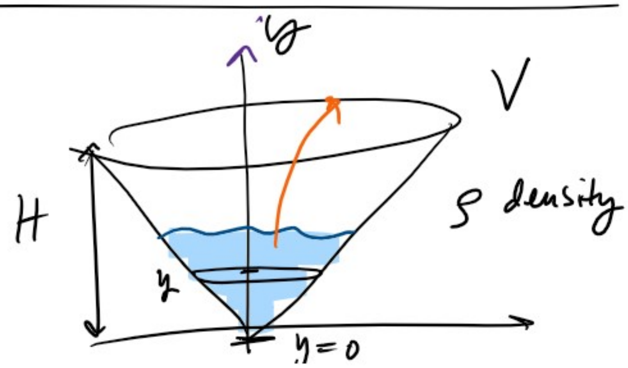
(c) $V = \int_0^1 2\pi x\left[(x+1) - (x+1)^2\right]dx$



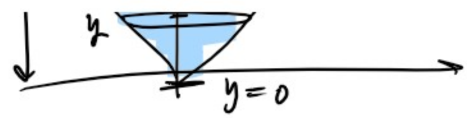
$r = x + 1$ $\quad$ $h = x - x^2$

Applications – Pumping

Work to pump water to the top of tank



the top of ...



$W = m \cdot g \cdot d$

where

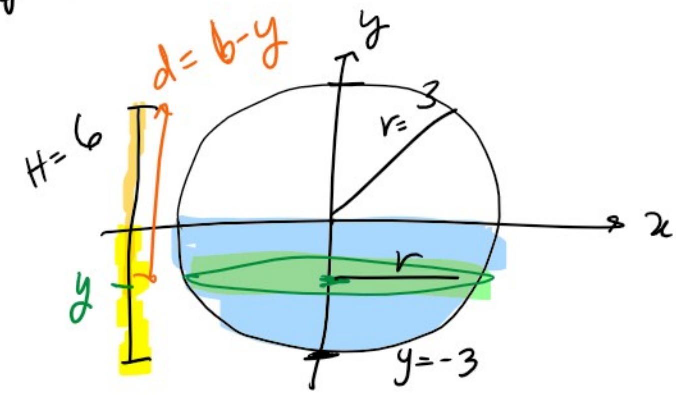
$m = \rho V \leftarrow$ integrate to find V

$d = H - y$

m – mass
g – grav. acc.
d – distance

Ex: S18 – F.E. – #5

A spherical tank 6ft in diameter is half full of water (ρ)
Find the work to pump all the water to the top of the tank



$W = m \cdot g \cdot d$

$m = \rho V$
$= \rho(\pi r^2)\Delta y$
$= \rho\pi(9-y^2)\Delta y$

$r = f(y) = \sqrt{3^2 - y^2}$

$r^2 = 9 - y^2$

$d = 6 - y$

$$W = \int_{-3}^{0} \rho\pi(9-y^2)\, g\,(6-y)\, dy$$

$$= \pi\rho g \int_{-3}^{0} (9-y^2)(6-y)\, dy$$

Vectors: let $\vec{u} = \langle u_1, u_2, u_3 \rangle$
$\vec{v} = \langle v_1, v_2, v_3 \rangle$

Dot Product: $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

$\vec{u} \cdot \vec{v} = 0 \Rightarrow \vec{u} \perp \vec{v}$

Cross Product: $\vec{w} = \vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$\vec{w} \perp \vec{u}$ and $\vec{v}$

if $\vec{u} \times \vec{v} = \vec{0} \Rightarrow \vec{u} \parallel \vec{v}$

Projections: $\text{proj}_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \right) \vec{v}$

area of parallelogram with sides $\vec{u}, \vec{v}$

$A = |\vec{u} \times \vec{v}|$

area of triangle $A = \frac{1}{2} |\vec{u} \times \vec{v}|$

length / magnitude: $|\vec{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2}$

unit vector: $\frac{\vec{u}}{|\vec{u}|}$

Ex: 517 - F.E. #2