Announcements:

Final Exam - Mon May 2nd @ 10:30 am - 12:30pm in ECLT

Office Hours: Friday @ 11am - noon on Zoom
@ 4 - 5pm in MATH 817

Warm Up: Find the arc length of the circle $r = 6\cos\theta$ for $0 \le \theta \le \frac{\pi}{3}$

(a) 2π
(b) 6π
(c) 4π
(d) 3π

$$L = \int_0^{\frac{\pi}{3}} \sqrt{f(\theta)^2 + f'(\theta)^2}\, d\theta$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{(6\cos\theta)^2 + (-6\sin\theta)^2}\, d\theta$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{6^2(\underbrace{\cos^2\theta + \sin^2\theta}_{=1})}\, d\theta$$

$$= \int_0^{\frac{\pi}{3}} 6\, d\theta = \left[6\theta\right]_0^{\frac{\pi}{3}} = 6\left(\frac{\pi}{3} - 0\right) = 2\pi$$

Exam 3: Benchmark

→ Booklets will be returned tomorrow in REC

→ Answer key will be posted

Final Exam: 120 minutes
25 Qs × 8 pts = 200 pts

cumulative — Lessons 1 - 34

Breakdown:

7 Qs – Exam 1 – Less 1-9 – Vectors / Areas + Volumes

6 Qs – Exam 2 – Less 10-18 – Integration Tech.

9 Qs – Exam 3 – Less 19-31 – Series + power series

3 Qs – Polar Coords – Less 32-34

NOT Covered:

- center of mass / centroid
- parametric eqns
- complex numbers

Same assigned seats

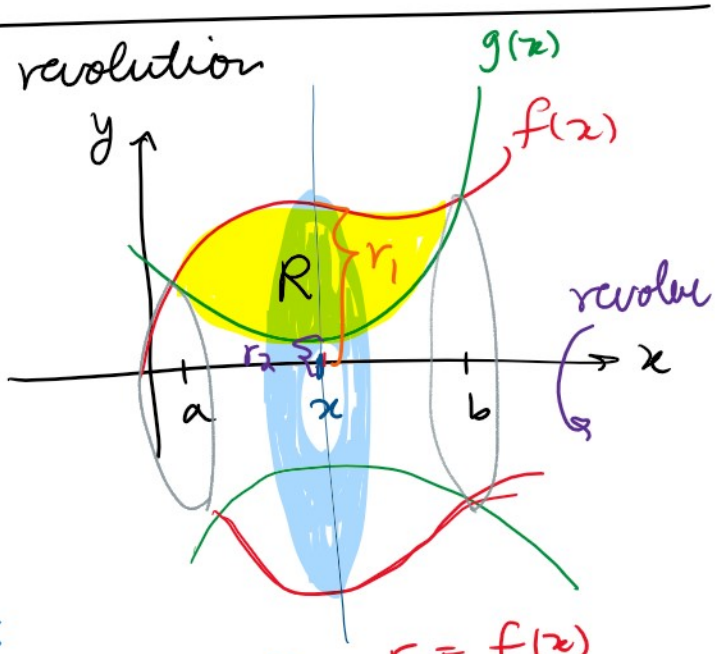
LEC Brightspace → Gradebook

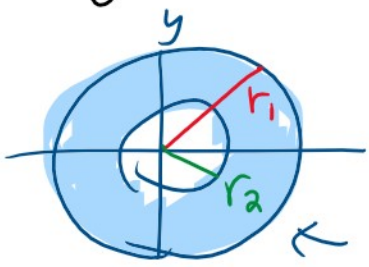
Equation List Given:

Finding Volumes of solids of revolution

Washer Method

$$V = \int_a^b \pi \left[f(x)^2 - g(x)^2 \right] dx$$



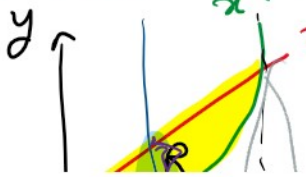


$A = \pi r_1^2 - \pi r_2^2$

$r_1 = f(x)$

$r_2 = g(x)$

HOTSEAT: Let R be the region between $y = x$, $y = x^2$, $x = 0$, $x = 1$



$y = x^2$, $x = 0$, $x = 1$

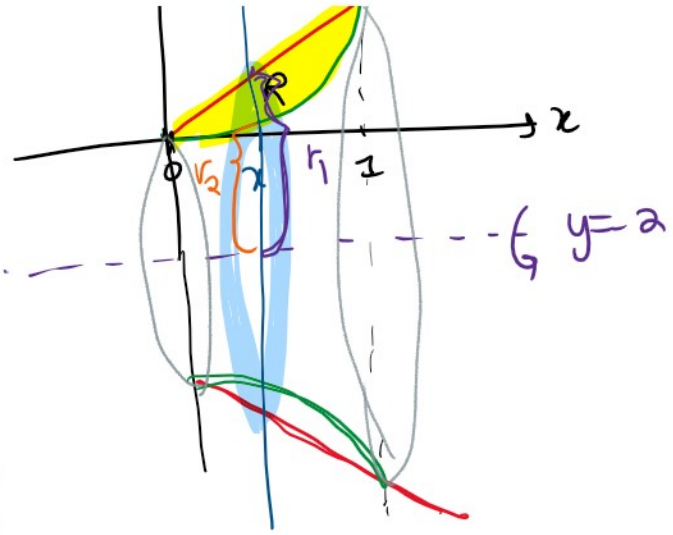
Revolve around the line $y = -2$

Use Washer Method to set the integral for V

(a) $V = \pi \int_0^1 (x-2)^2 - (x^2-2)^2 \, dx$

(b) $V = \pi \int_0^1 (x+2)^2 - (x^2+2)^2 \, dx$

(c) $V = \pi \int_0^1 x^2 - x^4 \, dx$



$r_1 = x + 2$

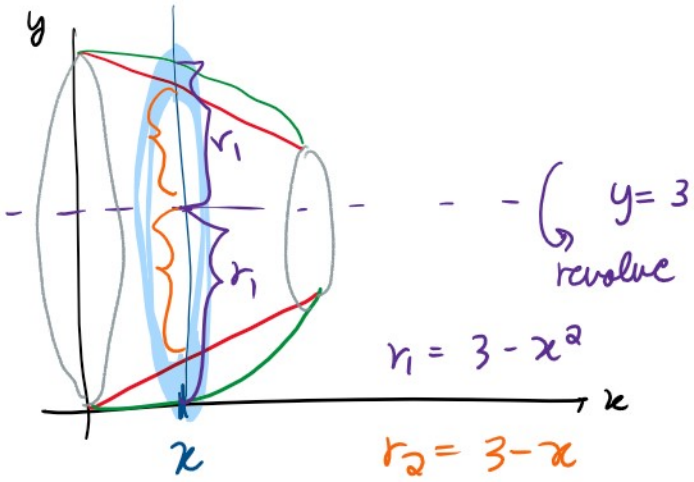
$r_2 = x^2 + 2$

HOTSEAT: Same R, revolve around $y = +3$

(a) $V = \pi \int_0^1 (3-x^2)^2 - (3-x)^2 \, dx$

(b) $V = \pi \int_0^1 (3+x^2) - (3-x)^2 \, dx$

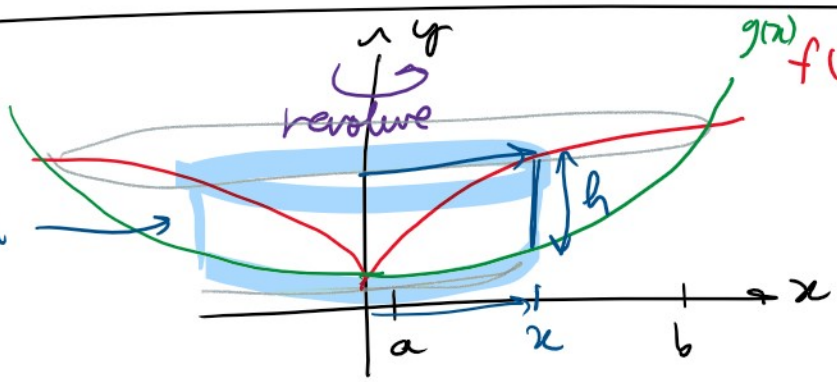
(c) $V = \pi \int_0^1 x^2 - x^4$



$r_1 = 3 - x^2$

$r_2 = 3 - x$

Shell Method



$A = 2\pi r \cdot h$

$h = f(x) - g(x)$

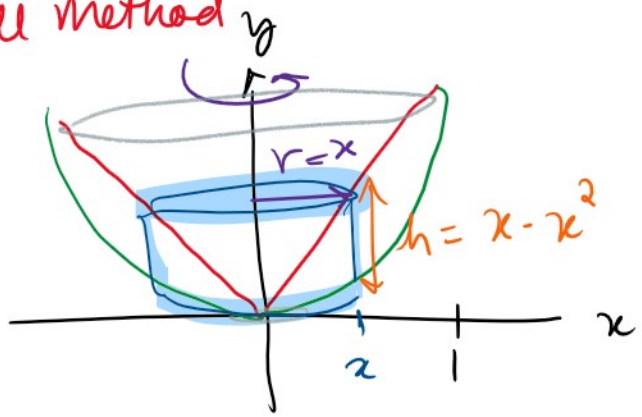
$r = x$

$$V = \int_a^b 2\pi x \left(f(x) - g(x) \right) dx$$

HOTSEAT: R region bdd by $y = x$, $y = x^2$, $x = 0$, $x = 1$

revolve around the y-axis.

Set up the integral using Shell Method

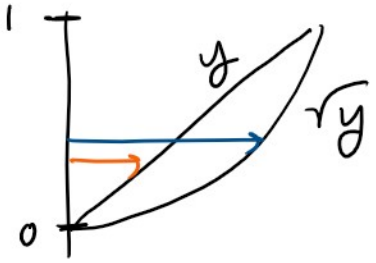


(a) $V = \int_0^1 2\pi x \, (x) \, dx$

(b) $V = \int_0^1 2\pi x \left[x^2 - x \right] dx$

(c) $V = \int_0^1 2\pi x \left[x - x^2 \right] dx$

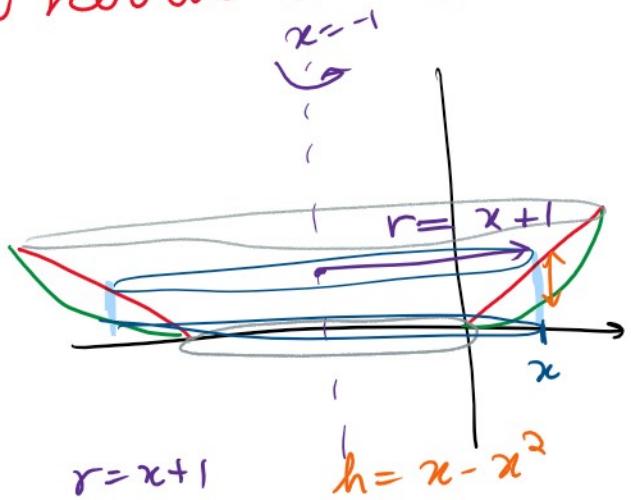
$r = x$



Washer

$$V = \int_0^1 \pi \left[(ry)^2 - (y)^2 \right] dy$$

HOTSEAT: Same R, now revolve around $x = -1$



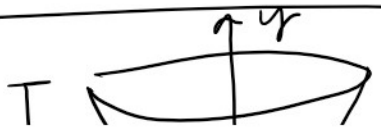
(a) $V = \int_0^1 2\pi x \left[x - x^2 \right] dx$

(b) $V = \int_0^1 2\pi (x+1) \left[x - x^2 \right] dx$

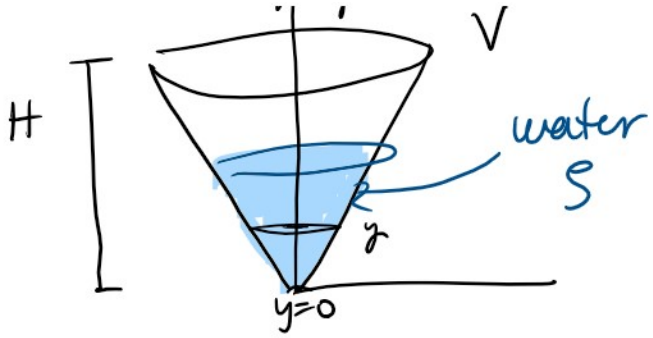
(c) $V = \int_0^1 2\pi x \left[(x+1) - (x+1)^2 \right] dx$

$r = x + 1$ $\quad h = x - x^2$

Applications – Pumping



Applications – Pumping



Work to pump water to the top of tank

$$W = m \cdot g \cdot d$$

where $m = \rho V$ ← of fluid ← integrate to find V

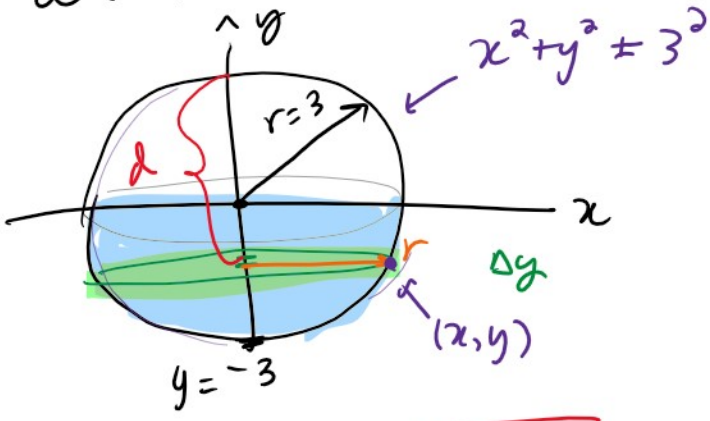
$$d = H - y$$

m – mass
g – grav acc.
d – distance

Ex: S18 – F.E.–#5

A spherical tank 6ft in diameter is half full of water. (density ρ)

Find the work to pump all the water to the top of the tank



$$W = m \cdot g \cdot d$$

$$m = \rho V = \rho(\pi r^2)\Delta y$$

$$= \rho\pi(9-y^2)\Delta y$$

$r = f(y) = x = \sqrt{3^2 - y^2}$

$r^2 = 9 - y^2$

$$d = 3 - y$$

$$W = \int_{-3}^{0} \rho\pi(9-y^2)\, g\,(3-y)\, dy$$

$$= \pi \rho g \int_{-3}^{0} (9 - y^2)(3 - y)\, dy$$