

**Problem 1**

(4 points)

This practice exam is provided as a way for students to practice solving the exam under a time limit. We suggest that you set a timer for 60 minutes, put away your notes and calculator, and solve the exam in one sitting.

On the real exam, you will earn 4 points for correctly filling out the scantron. The real exam is multiple choice graded all-or-nothing. (In this practice exam, ignore any prompts to show your work. While showing your work is good practice, on the exam it will not be graded.)

Answer = \_\_\_\_\_

**Problem 2**

(8 points)

Find the minimum value of  $f(x,y) = 2x + 3y + 2$  given that  $2x^2 + 5xy + 4y^2 = 28$

To receive the full 5 points, you must show all your work on this problem.

- A. -3
- B. -8
- C. -2
- D. -1
- E. -6

**Problem 3**

(8 points)

Evaluate  $\iint_R \frac{x^2 y}{2 + x^3} dA$  over the region  $R = \{(x,y): 1 \leq x \leq 2, 0 \leq y \leq 4\}$ .

- A.  $\frac{8}{3} (\ln(10) - \ln(3))$
- B.  $\frac{8}{3} (\ln(5) - \ln(1.5))$
- C.  $\ln(33)$
- D.  $\frac{8}{3} (\arctan(5) - \arctan(1.5))$
- E.  $8 \left( \frac{7}{3} + \ln(2) \right)$

**Problem 4**

(8 points)

By changing the order of integration, compute  $\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-y^2} dy dx$

- A.  $2/3$
- B.  $\pi/4$
- C.  $1$
- D.  $1/3$
- E.  $0$

**Problem 5**

(8 points)

Let  $D$  be the region in the **first quadrant** between the circles  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 4$ . Evaluate the

integral  $\iint_D \frac{x^2 y}{(x^2 + y^2)^{3/2}} dA.$

- A.  $14/3$
- B.  $5/6$
- C.  $10/3$
- D.  $1/2$
- E.  $3/2$

**Problem 6**

(8 points)

Consider the tetrahedron  $E$  with vertices  $(0,0,0)$ ,  $(1,0,0)$ ,  $(0,2,0)$ ,  $(0,0,3)$ . Express  $\iiint_E x dV$  as an iterated integral in the order  $dzdydx$ .

- A.  $\int_0^1 \int_0^{2-2x} \int_0^{-3x - \frac{3}{2}y + 3} x dz dy dx$
- B.  $\int_0^1 \int_0^{2-2x} \int_0^{-3x + \frac{3}{2}y + 3} x dz dy dx$
- C.  $\int_0^1 \int_0^{2-2x} \int_0^{3x - \frac{3}{2}y - 3} x dz dy dx$
- D.  $\int_0^1 \int_0^{2-2x} \int_0^{3x + \frac{3}{2}y - 3} x dz dy dx$
- E.  $\int_0^1 \int_0^{2-2x} \int_0^{-3x - \frac{3}{2}y - 3} x dz dy dx$

**Problem 7**

(8 points)

The triple integral

$$\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{x^2+y^2}} 8(x^2 + y^2) dz dy dx$$

when converted to cylindrical coordinates becomes

- A.  $\int_0^\pi \int_0^9 \int_0^r 8r^2 dz dr d\theta$
- B.  $\int_0^\pi \int_0^3 \int_0^r 8r^3 z dz dr d\theta$
- C.  $\int_0^\pi \int_0^9 \int_0^r 8r^2 dz dr d\theta$
- D.  $\int_0^\pi \int_0^3 \int_0^r 8r^2 z dz dr d\theta$
- E.  $\int_0^\pi \int_0^3 \int_0^r 8r^3 dz dr d\theta$

**Problem 8**

(8 points)

Convert the integral to spherical coordinates and compute it:

$$\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} 3 dz dy dx$$

- A.  $10(\sqrt{2} - 1)\pi$
- B.  $12(\sqrt{2} - 1)\pi$
- C.  $16(\sqrt{2} - 1)\pi$
- D.  $8(\sqrt{2} - 1)\pi$
- E.  $2(\sqrt{2} - 1)\pi$

**Problem 9**

(8 points)

A lamina with density  $\rho(x,y) = xy$  occupies the region of the plane bounded by  $y = x^2$ ,  $y = 1$  and  $x = 0$ . The mass of lamina is equal to  $\frac{1}{6}$ . Find the y-coordinate of its center of mass.

- A. 3/4
- B. 5/6
- C. 7/8
- D. 12/21
- E. 2/3

**Problem 10**

(8 points)

Let  $f(x,y,z) = x^2 + xy + z^4 - z$  and let  $(a,b,c)$  be a point where  $\nabla f(a,b,c) = \langle 3, 5, -5 \rangle$ . Find the value of  $a + b - c$ .

- A. 0
- B. -3
- C. -2
- D. 1
- E. -1

**Problem 11**

(8 points)

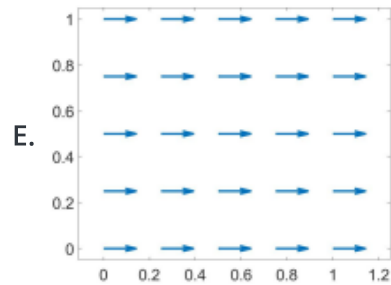
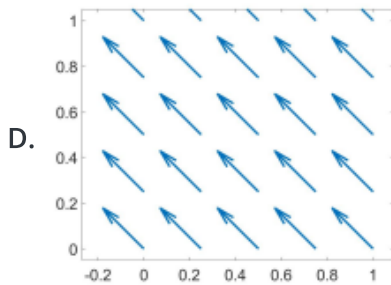
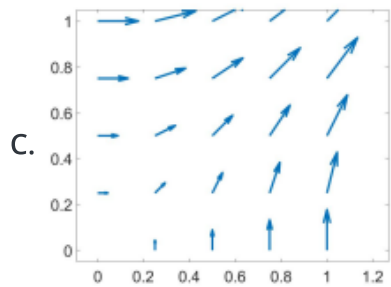
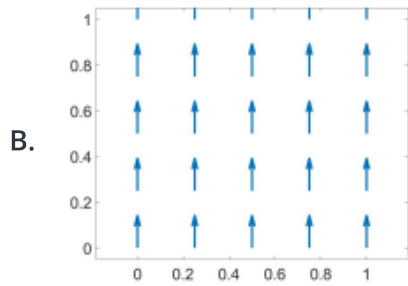
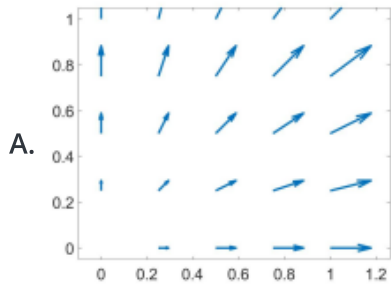
Compute the line integral  $\int_C (4x^3 + y^3) ds$ , where  $C$  is the line segment from  $(0,0)$  to  $(1,2)$ .

- A.  $\frac{5\sqrt{5}}{4}$
- B.  $-5$
- C.  $\sqrt{5}\pi$
- D. 0
- E.  $3\sqrt{5}$

**Problem 12**

(8 points)

A particle is traveling on the path  $y = x$  from  $(0,0)$  to  $(1,1)$ . For which of the following force vector fields is the work done equal to 0?



CORRECTION:

$$x^2 - 3y^2$$

**Problem 13**

(8 points)

If  $\vec{F} = (3 + 2xy)\vec{i} + (x^2 - 3y)\vec{j}$  and  $\vec{F} = \nabla f$ , find  $\int_C \nabla f \cdot d\vec{r}$  if the curve  $C$  is parametrized as  $\vec{r}(t) = e^t \sin(t)\vec{i} + e^t \cos(t)\vec{j}$ ,  $0 \leq t \leq \pi$ .

- A.  $e^{3\pi} + 1$
- B.  $\pi^3$
- C. 0
- D.  $-e^{3\pi} - 1$
- E.  $-\pi^3$

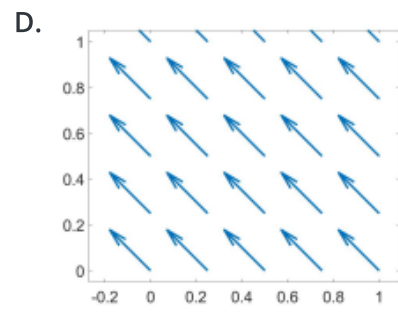
## Answer Key

| PROBLEM | ANSWER   |
|---------|--|
| 1.      | <i>Name</i>  |
| 2.      | E. -6  |
| 3.      | A. $\frac{8}{3}(\ln(10) - \ln(3))$                                 |
| 4.      | A. $2/3$   |
| 5.      | D. $1/2$   |
| 6.      | A. $\int_0^1 \int_0^{2-2x} \int_0^{-3x-\frac{3}{2}y+3} x dz dy dz$ |
| 7.      | E. $\int_0^\pi \int_0^3 \int_0^r 8r^3 dz dr d\theta$               |
| 8.      | C. $16(\sqrt{2} - 1)\pi$   |
| 9.      | A. $3/4$   |
| 10.     | E. -1  |
| 11.     | E. $3\sqrt[3]{5}$  |

PROBLEM

ANSWER

12.



13.

A.  $e^{3\pi} + 1$